Transmitter and Receiver Processing in Block Transmission Systems With and Without Guard Periods

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Abstract — **Block transmission systems with guard periods are used for high rate transmissions over wireless channels. Removing the guard periods would increase the throughput at the price of higher computational complexity. Here a method called overlapping is presented that retains the algebraic structure of the block processing stages while the insertion of guard periods can be omitted. The necessary computations of the overlapping method can be performed in the receiver. If they are transfered to the transmitter side, a new system with block wise pre equalization and optional error correction in the receiver is obtained.**

I. Introduction

The application of block transmission techniques enable high data rate transmissions in time dispersive wireless channels with reasonable computational complexity. If the particular blocks are not separated by a guard period interference between successive blocks (inter block interference: IBI) will degrade the performance of the transmission system substantially. Different types of guard periods are known, e.g., the cyclic prefix (CP), the zero pads (ZP) and the pseudo noise training sequence (PN) [2, 12]. The guard periods remove the IBI, however they cause throughput loss. In existing block transmission systems such as the wireless LAN IEEE 802.11a 20 per cent of the transmission time are allocated to the cyclic prefix. A larger block length would decrease the throughput loss caused by the guard period, however, it would increase the processing delay, the storage requirements and the processor power. Furthermore the longer the block length the more unrealistic becomes the assumption of the time invariance of the channel during one block. Thus the maximum length of one block is restricted. Considering this background methods that retain the block processing structure of the transmission systems while not necessarily insert guard periods are of great interest. Block transmission systems without guard periods are considered in [5, 8, 9].

In this article block transmission systems with and without guard periods are considered. Several blocks compose one data burst, that is transmitted over a wireless channel. The channel is assumed to be time invariant during one burst and exactly known. Block transmission systems, that use either a cyclic prefix (CP) or zero pads (ZP) as guard period are initially reviewed. A technique which transfers the ZP method to the CP method is presented. This is important to enable common transceiver processing in the CP and ZP based transmission systems. Each processing stage of the block transmission systems is described by a matrix operation. Sometimes the order of the matrices can be changed. This means that the order of the processing stages in a transmission system can be changed too. The question which processing stage can be performed in the transmitter or in the receiver is answered by the succession of the matrices. The different partitioning of the stages in transmitter and receiver processing leads to different transmission systems which show significant performance differences. Afterwards methods that avoid the guard period while keeping the block processing structure and the specific computations of the different processing stages are presented. They are called overlapping methods. Then it is analyzed which operations of the overlapping method can be performed in the transmitter and in the receiver, respectively. This leads to a new block transmission system without guard periods. It is based on equalization at the transmitter side and optional error correction at the receiver side.

First block transmission systems are reviewed in chapter II. Then the overlapping methods are presented in chapter III. Based on the partitioning in transmitter and receiver processing a transmission system with block wise pre equalization and error correction that does not require guard periods is presented. Finally some conclusions are drawn.

II. Block Transmission Systems

If a data burst $\mathbf{d}$ is transmitted over a time invariant time dispersive channel characterized by its channel vector $\mathbf{h}$, the received vector $\mathbf{x}$ can be computed by convolving the channel with the data and by adding a noise vector $\mathbf{n}$:

$$\mathbf{x} = \mathbf{H}\mathbf{d} + \mathbf{n}. \quad (1)$$

Instead of performing channel equalization by an FIR filter (multiply $\mathbf{x}$ by a Toeplitz matrix that contains filter weights to obtain estimates $\hat{\mathbf{d}}$ of the data $\mathbf{d}$) the transmitted data $\mathbf{d}$ can be estimated from the received vector $\mathbf{x}$ and the channel vector $\mathbf{h}$ by solving the system of equation. If the solution is performed block wise the transceiver complexity, the processing delay, the storage requirements and the processing power is decreased. To enable block wise processing the channel matrix $\mathbf{H}$ can be decomposed into independent blocks of channel matrices by one of the following methods: the cyclic prefix method, the zero padding method, or the pseudo noise method.

Subsequently the cyclic prefix method is described. For it two parameters need to be introduced, the length of one data block B and the length of the channel vector L. A cyclic prefix is inserted in one data block $\mathbf{d}_b \in \mathbb{C}^B$ by multiplication from left with the matrix

$$\mathbf{P}_{CB} = \begin{bmatrix} \mathbf{O}_{(L-1)\times(B+L-1)} & \mathbf{I}_{L-1} \\ \multicolumn{2}{c}{\mathbf{I}_B} \end{bmatrix}.$$

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Here the length of the cyclic prefix is set to $L-1$. This is its minimum length to avoid interference between subsequent data blocks. After transmitting the data block $\mathbf{d}_b$ extended by a cyclic prefix over the channel the cyclic prefix is discarded by multiplying from left with the matrix

$$\mathbf{P}_{DB} = \left[\begin{array}{cc} \mathbf{O}_{B\times(L-1)} & \mathbf{I}_B \end{array} \right].$$

Next a square block $\mathbf{H}_B \in \mathbb{C}^{(B+L-1)\times(B+L-1)}$ of the channel matrix $\mathbf{H}$ (see equation 1) that corresponds to one cyclic extended data block $\mathbf{d}_b$ is considered. Applying the cyclic prefix method to this block means multiplying $\mathbf{P}_{CB}$ from right and $\mathbf{P}_{DB}$ from left. Then a circulant block $\tilde{\mathbf{H}}_B \in \mathbb{C}^{B\times B}$ is obtained:

$$\tilde{\mathbf{H}}_B = \mathbf{P}_{DB}\mathbf{H}_B\mathbf{P}_{CB}.$$

Assuming the data burst $\mathbf{d}$ consists of J blocks of size B, equation 1 can be reformulated by using the cyclic prefix method. Each block $\mathbf{d}_b$ of the data burst $\mathbf{d} \in \mathbb{C}^{JB}$ is processed in the afore mentioned fashion. This can be denoted by using the Kronecker product. The matrices $\mathbf{P}_C$ and $\mathbf{P}_D$ that frequently insert and discard the cyclic prefix in the data burst $\mathbf{d}$ are then given by:

$$\mathbf{P}_C = \mathbf{I}_J \otimes \mathbf{P}_{CB}, \quad \text{and} \quad \mathbf{P}_D = \mathbf{I}_J \otimes \mathbf{P}_{DB}.$$

If $\mathbf{H}_C \in \mathbb{C}^{J(B+L-1)\times J(B+L-1)}$ is the channel matrix corresponding to the cyclic prefixed data burst reduced by the last $L-1$ rows the following equation can be established:

$$\mathbf{I}_J \otimes \tilde{\mathbf{H}}_B = \mathbf{P}_D\mathbf{H}_C\mathbf{P}_C.$$

This means, performing the cyclic prefix method is equivalent to transforming the large channel matrix $\mathbf{H}_C$ into small independent circulant blocks $\tilde{\mathbf{H}}_B$. After applying the cyclic prefix method to the system of equations 1 the circulant block system is obtained:

$$\begin{aligned} \mathbf{x}_C &= \mathbf{P}_D\mathbf{H}_C\mathbf{P}_C\mathbf{d} + \mathbf{n}_C \\ &= \mathbf{I}_J \otimes \tilde{\mathbf{H}}_B\mathbf{d} + \mathbf{n}_C. \end{aligned} \qquad (2)$$

The data $\hat{\mathbf{d}}$ is detected by solving this system of equations. Since the blocks do not interfere with each other they can be processed independently. Data estimates of one block $\hat{\mathbf{d}}_B$ are obtained by computing the inverse of $\tilde{\mathbf{H}}_B$ and multiplying it to the corresponding part of $\mathbf{x}_C$. The inverse can be computed by applying different methods, e. g., least square, minimum mean square error and other criteria.

In view of an efficient implementation the eigenvalue decomposition of the circulant block $\tilde{\mathbf{H}}_B$ and of its inverse $\tilde{\mathbf{H}}_B^{-1}$ plays an important role. It is given by

$$\tilde{\mathbf{H}}_B = \mathbf{F}_B^{-1}\mathbf{D}_B\mathbf{F}_B, \text{ and } \tilde{\mathbf{H}}_B^{-1} = \mathbf{F}_B^{-1}\mathbf{D}_B^{-1}\mathbf{F}_B.$$

The matrix $\mathbf{F}_B$ is the DFT matrix of size B. They can be implemented efficiently by using the FFT. The diagonal matrix $\mathbf{D}_B$ contains the eigenvalues of $\tilde{\mathbf{H}}_B$. They can be computed by transforming (DFT) the channel vector zero padded to size B, which is equal to the first column of $\tilde{\mathbf{H}}_B$:

$$\mathbf{D}_B = \text{diag}(\mathbf{F}\tilde{\mathbf{H}}_B(:,1)).$$

Here diag($\mathbf{b}$) is a diagonal matrix with the elements of $\mathbf{b}$ on its diagonal. Extending the decomposition of one block $\tilde{\mathbf{H}}_B$ to all J blocks the following block matrices are obtained:

$$\mathbf{I}_J \otimes \tilde{\mathbf{H}}_B = \mathbf{F}^{-1}\mathbf{D}\mathbf{F} \text{ and } (\mathbf{I}_J \otimes \tilde{\mathbf{H}}_B)^{-1} = \mathbf{F}^{-1}\mathbf{D}^{-1}\mathbf{F},$$
$$\text{where } \mathbf{F} = \mathbf{I}_J \otimes \mathbf{F}_B \text{ and } \mathbf{D} = \mathbf{I}_J \otimes \mathbf{D}_B.$$

Applying the cyclic prefix method and the eigenvalue decomposition of circulant block matrices four methods to compute the estimates $\hat{\mathbf{d}}$ of the data $\mathbf{d}$ can be found. Some of the necessary computations are performed in a transmitter others are performed in the receiver:

$$\hat{\mathbf{d}}_{\text{OFDM}} = \underbrace{\mathbf{D}^{-1}\mathbf{F}\mathbf{P}_D}_{\text{receiver}}\mathbf{H}_C\underbrace{\mathbf{P}_C\mathbf{F}^{-1}\mathbf{d}}_{\text{transmitter}} + \mathbf{D}^{-1}\mathbf{F}\mathbf{P}_D\mathbf{n}_C.$$
$$\hat{\mathbf{d}}_{\text{OFDMPRE}} = \underbrace{\mathbf{F}\mathbf{P}_D}_{\text{receiver}}\mathbf{H}_C\underbrace{\mathbf{P}_C\mathbf{F}^{-1}\mathbf{D}^{-1}\mathbf{d}}_{\text{transmitter}} + \mathbf{F}\mathbf{P}_D\mathbf{n}_C.$$
$$\hat{\mathbf{d}}_{\text{CPSC}} = \underbrace{\mathbf{F}^{-1}\mathbf{D}^{-1}\mathbf{F}\mathbf{P}_D}_{\text{receiver}}\mathbf{H}_C\underbrace{\mathbf{P}_C\mathbf{d}}_{\text{transmitter}} + \mathbf{F}^{-1}\mathbf{D}^{-1}\mathbf{F}\mathbf{P}_D\mathbf{n}_C.$$
$$\hat{\mathbf{d}}_{\text{CPSCPRE}} = \underbrace{\mathbf{P}_D}_{\text{receiver}}\mathbf{H}_C\underbrace{\mathbf{P}_C\mathbf{F}^{-1}\mathbf{D}^{-1}\mathbf{F}\mathbf{d}}_{\text{transmitter}} + \mathbf{P}_D\mathbf{n}_C.$$

The resulting transmission schemes that inherently solve the system of equations 2 are known as OFDM (Orthogonal Frequency Domain Multiplexing), CP-SC (Cyclic Prefix based Single Carrier), pre equalized OFDM, and pre equalized CP-SC. The systems are all suited for high data rate transmissions over wireless channels, since they remove the interference caused by the channel very efficiently with exactly the same amount of computations. Nevertheless, there are some significant differences: the channel estimates need to be known either in the transmitter or in the receiver, the enhancement of the noise, the peak to average ratio of the transmitted signal, the feasibility of adaptive modulation in time or frequency domain, the receiver and transmitter complexity, and the similarity of the transmitter and receiver processing. Further information about the systems can be found in [1, 7, 8, 12].

Next we focus on the zero padding method. Zero pads are inserted into one data block $\mathbf{d}_b \in \mathbb{C}^B$ by multiplying from left by

$$\mathbf{P}_{ZB} = \left[\begin{array}{c} \mathbf{I}_B \\ \mathbf{0}_{(L-1)\times B} \end{array} \right].$$

To avoid inter block interference the length of the inserted zero vector is at least of the length $L-1$. The matrix that frequently inserts zeros in the data burst $\mathbf{d} \in \mathbb{C}^{JB}$ is given by:

$$\mathbf{P}_Z = \mathbf{I}_J \otimes \mathbf{P}_{ZB}.$$

If the channel convolution matrix $\mathbf{H}_T$ of size $J(B+L-1)+L-1$ by $J(B+L-1)$ is multiplied from left by $\mathbf{P}_Z$ the following identity is obtained:

$$\mathbf{H}_T\mathbf{P}_Z = \mathbf{I}_J \otimes \mathbf{H}_{TB}, \text{ where } \mathbf{H}_{TB} \in \mathbb{C}^{(B+L-1)\times B}$$

The system of equations 1 after applying the zero padding method is transformed to:

$$\begin{aligned} \mathbf{x}_T &= \mathbf{H}_T\mathbf{P}_Z\mathbf{d} + \mathbf{n}_T \\ &= \mathbf{I}_J \otimes \mathbf{H}_{TB}\mathbf{d} + \mathbf{n}_T. \end{aligned} \qquad (3)$$

Estimates $\hat{\mathbf{d}}$ of the data $\mathbf{d}$ are obtained by solving this system of equations. Since the blocks are independent, each block can be processed independently. Computing the pseudo inverse $\mathbf{H}_{TB}^{+}$ of $\mathbf{H}_{TB}$ and multiplying it to each corresponding block $\mathbf{x}_{Tb}$ of $\mathbf{x}_T$ render the estimate of $\mathbf{d}$. Since $\mathbf{H}_{TB}$ has different properties than $\tilde{\mathbf{H}}_B$, e.g., the existence of the pseudo inverse is guaranteed, and the matrix $\mathbf{H}_{TB}$ averages a better condition (means lower noise enhancement) than $\tilde{\mathbf{H}}_B$ [10], we expect an improved quality of the estimates compared to the post equalized cyclic method.

The pseudo inverse $\mathbf{H}_{TB}^{+}$ can be computed from $\mathbf{H}_{TB}$ by the following equation: $\mathbf{H}_{TB}^{+} = (\mathbf{H}_{TB}^{H}\mathbf{H}_{TB})^{-1}\mathbf{H}_{TB}^{H}$. Since $\mathbf{H}_{TB}^{H}\mathbf{H}_{TB}$ is a symmetric positive definite matrix, the Cholesky decomposition [4] exists: $\mathbf{H}_{TB}^{H}\mathbf{H}_{TB} = \mathbf{R}_{TB}^{H}\mathbf{R}_{TB}$. The upper triangular matrix $\mathbf{R}_{TB}$ is called Cholesky factor. It keeps the band structure of $\mathbf{H}_{TB}^{H}\mathbf{H}_{TB}$. It can be computed by the Cholesky algorithm [4] or, computational more efficient, by an approximated computation [11]. One estimated data block $\hat{\mathbf{d}}_b$ is obtained by solving the equation $\mathbf{R}^{H}\mathbf{R}\hat{\mathbf{d}}_b = \mathbf{H}_{TB}^{H}\mathbf{x}_{Tb}$. For it two back substitutions and one matrix vector product with the banded Toeplitz matrix $\mathbf{H}_{TB}^{H}\mathbf{x}_{Tb}$ is performed. Estimates of the whole burst $\mathbf{d}$ are obtained by solving the system of equations 3 block wise, that means multiplying the received data $\mathbf{x}_T$ from left by the pseudo inverse

$$\mathbf{H}_{T2}^{+} \quad = \quad \mathbf{I}_J \otimes \mathbf{H}_{TB}^{+} = \mathbf{I}_J \otimes \left[\mathbf{R}^{-1}\mathbf{R}^{-H}\mathbf{H}_{TB}^{H}\right].$$

This leads to the following transmission scheme:

$$\begin{aligned}\hat{\mathbf{d}}_{\mathrm{ZPSC}} &= \mathbf{H}_{T2}^{+}\mathbf{x}_T,\\ &= \underbrace{\mathbf{H}_{T2}^{+}}_{\text{receiver}}\ \mathbf{H}_T\ \underbrace{\mathbf{P}_Z\mathbf{d}}_{\text{transmitter}} + \mathbf{H}_{T2}^{+}\mathbf{n}_T\\ &\text{where } \mathbf{H}_{T2}^{+}\mathbf{H}_T\mathbf{P}_Z = \mathbf{I}.\end{aligned}$$

This transmission scheme that inherently solves the system of equations 3 is called ZP-SC (Zero Padded Single Carrier). ZP-SC needs fewer transmit power and can detect with lower detection errors than post equalized CP systems (OFDM, CP-SC). Further information of the system can be found in [12]. Although the back substitutions can also be performed at the transmitter side we can not perform the multiplication with the pseudo inverse in the transmitter. This is because the matrix $\mathbf{H}_{TB}$ has full colum rank if the channel vector is not equal to the zero vector. If $\mathbf{H}_{TB}$ has full column rank there exists a matrix $\mathbf{H}_{TB}^{+}$ with $\mathbf{H}_{TB}^{+}\mathbf{H}_{TB} = \mathbf{I}_B$. The matrix $\mathbf{H}_{TB}^{+}$ is called the pseudo inverse of $\mathbf{H}_{TB}$. However, we can not find an arbitrary matrix $\mathbf{B}$ multiplied from the right to matrix $\mathbf{H}_{TB}$, so that $\mathbf{H}_{TB}\mathbf{B} = \mathbf{I}_{B+L-1}$. The rank of $\mathbf{H}_{TB}$ is determined by the number of its columns. Multiplying $\mathbf{B}$ from the right computes several linear combinations of the columns of $\mathbf{H}_{TB}$. Thus $\mathbf{H}_{TB}\mathbf{B}$ has the maximum rank of B. However, the identity matrix $\mathbf{I}_{B+L-1}$ has always full rank. Matrices with different rank can not be equal.

However, there exists a technique to transfer the zero padding method to the cyclic prefix method. Then zero padded block transmissions can be detected by using the four transmission systems known from the cyclic prefix method. Thus common architectures for CP and ZP based transmission systems become feasible.

The technique to transfer the ZP method to the CP method is now described. A matrix $\mathbf{P}_{AB}$ which adds the last $L-1$ entries of a received block to the first and a matrix $\mathbf{P}_B$ which does it frequently for the whole received data burst are introduced:

$$\begin{aligned}\mathbf{P}_{AB} &= \begin{bmatrix} \mathbf{I}_B & \begin{matrix}\mathbf{I}_{L-1}\\ \mathbf{0}_{(B-L+1)\times(L-1)}\end{matrix}\end{bmatrix},\\ \mathbf{P}_A &= \mathbf{I}_J \otimes \mathbf{P}_{AB}.\end{aligned}$$

The matrix $\mathbf{I}_J \otimes \tilde{\mathbf{H}}_B$ that consists of circulant blocks of size B can be obtain by applying the matrix $\mathbf{P}_A$ in the following fashion:

$$\mathbf{I}_J \otimes \tilde{\mathbf{H}}_B = \mathbf{P}_D\mathbf{H}_C\mathbf{P}_C = \mathbf{P}_A\mathbf{H}_T\mathbf{P}_Z.$$

If the cyclic prefix insertion matrix $\mathbf{P}_C$ is substituted by the zero padding matrix $\mathbf{P}_Z$ and additionally the discard matrix $\mathbf{P}_D$ is substituted by $\mathbf{P}_A$ a circulant block channel $\mathbf{I}_J \otimes \tilde{\mathbf{H}}_B$ is created. Thus the four transmission systems resulting from the CP method can also be applied to the ZP case. However, in both the CP and ZP based systems, the inserted guard periods result in throughput loss. In the next chapter a method that on the one hand retains the block wise processing and the algebraic structure of the processing stages and on the other hand can remove the guard periods is presented.

III. Overlapping Method

Overlapping is a method to invert a band matrix approximately and block wise. The application to the channel matrix $\mathbf{H}$ of equation 1 is depicted in figure 1. The large matrix $\mathbf{H}$ is divided into smaller blocks, which overlap each other. The overlapping blocks can be assumed to be either circulant or Toeplitz structured. The importance of the selected structure is analyzed in [10]. Then each small system of equations defined by the respective block is computed separately. Although the blocks are coupled we treat them independently. Thus only an approximate solution is obtained. In the cyclic case the submatrices are additionally assumed to be circulant although they are not. In this way some of the data are estimated twice or even more often. It is shown in [9] that the estimates in the middle part of one block have significant lower errors than at the edges. In the case where two or more estimates are computed the one which is closest to the middle of its respective data block is selected.

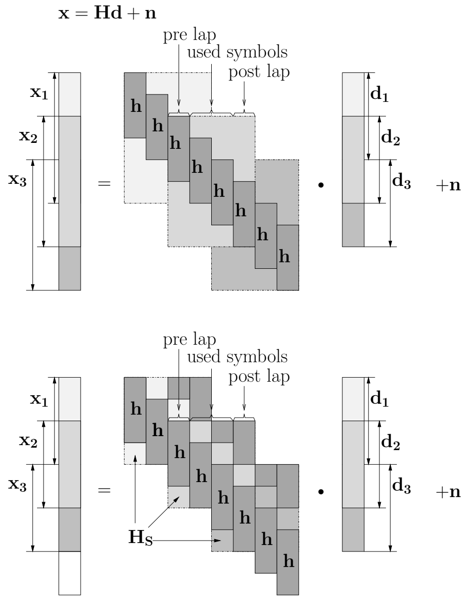

Figure 1: Circulant and Toeplitz structured submatrices.

Next the overlapping method with circulant submatrices is described. Some further parameters need to be introduced. These are the length of the pre lap P, the length of the post lap Q, the block size of one submatrix B_2 and the number of necessary submatrices $K = \lceil JB/(B_2 - P - Q)\rceil$ to obtain estimates for the whole data burst. The pre lap and the post lap determine the part of one estimated data block, which are not taken as final estimate. The submatrices overlap each other by $P+Q$ rows and columns. The transmitted and received data block corresponding to one submatrix $\mathbf{H}_S \in \mathbb{C}^{B_2\times B_2}$ are selected by the matrix $\mathbf{I}_{OL}$, which is computed by (MATLAB notation):

$B_{OL} = B_2 - P - Q;$
for $k = 1 : K$
$\mathbf{I}_{OL}((k-1)B_2 + 1 : KB_2,$
$(k-1)B_{OL} + 1 : (k-1)B_{OL} + B_2) = \mathbf{I}_{B_2};$
end;

With it we can establish the following system of equations:

$$\mathbf{I}_{OL}\mathbf{x} = [\mathbf{I}_K \otimes \mathbf{H}_S]\, \mathbf{I}_{OL}\mathbf{d} + \mathbf{I}_{OL}\mathbf{n}.$$

The matrix $\mathbf{H}_S$ denotes the selected submatrix. The solution renders the estimates of the overlapping data blocks $\mathbf{I}_{OL}\mathbf{d} = [\mathbf{I}_K \otimes \mathbf{H}_S]^{-1} \mathbf{I}_{OL}\mathbf{x}$. Then the first P (pre lap) and the last Q (post lap) values of each block are discarded. In other words, only the middle part of each estimated data block is considered. The selection out of one block is performed by a selector matrix $\mathbf{I}_{SB}$ and for all blocks by $\mathbf{I}_S$:

$$\begin{aligned} \mathbf{I}_{SB} &= \left[\begin{array}{ccc} \mathbf{0}_{B_{OL}\times P} & \mathbf{I}_{B_{OL}} & \mathbf{0}_{B_{OL}\times Q} \end{array} \right], \\ \mathbf{I}_S &= \mathbf{I}_K \otimes \mathbf{I}_{SB}. \end{aligned}$$

By using the selector matrix, the estimates $\hat{\mathbf{d}}_{OL}$ of the data burst $\mathbf{d}$ are obtained:

$$\hat{\mathbf{d}}_{OL} = \mathbf{I}_S\, [\mathbf{I}_K \otimes \mathbf{H}_S]^{-1}\, \mathbf{I}_{OL}\mathbf{x}.$$

Each of the selected submatrix should be approximately inverted by $\tilde{\mathbf{H}}_{B_2}^{-1} = \mathbf{F}_{B_2}^{-1}\mathbf{D}_{B_2}\mathbf{F}_{B_2}$. This can be performed from the right and from the left side. To invert all blocks approximately a further matrix is defined:

$$\mathbf{H}_{APP}^{-1} = \mathbf{I}_K \otimes \tilde{\mathbf{H}}_{B_2}^{-1}.$$

This will lead to the following approximation of inverting the channel $\mathbf{H}$ (see equation 1).

$$\mathbf{I}_S\mathbf{H}_{APP}^{-1}\mathbf{I}_{OL}\mathbf{H} \approx \mathbf{I}.$$

Subsequently we focus on the data detection error in the cases of post and pre equalization. Thereby the pre and post lap are set to zero ($P = Q = 0$). The system matrix $\mathbf{H}$ of equation 1 is identical to the sum of $\mathbf{H}_{APP}$ and an error matrix $\mathbf{H}_{ERR}$:

$$\mathbf{H} = \mathbf{H}_{APP} + \mathbf{H}_{ERR}.$$

The error matrix consists of matrix blocks

$$\mathbf{H}_{EB} = \begin{bmatrix} h_L & h_{L-1} & \cdots & h_2 \\ & h_L & \cdots & h_3 \\ & & \ddots & \vdots \\ & & & h_L \end{bmatrix}$$

that are frequently inserted into a matrix of zeros in the following fashion:

$$\begin{aligned} \mathbf{H}_{EB2} &= \begin{bmatrix} \mathbf{0} & -\mathbf{H}_{EB} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \in \mathbb{C}^{B_2 \times B_2}, \\ \mathbf{H}_{ERR1} &= \mathbf{I}_K \otimes \mathbf{H}_{EB2}, \\ \mathbf{H}_{ERR2} &= \begin{bmatrix} \mathbf{0}_{B_2 \times KB_2} & \\ -\mathbf{I}_{K-1} \otimes \mathbf{H}_{EB2} & \mathbf{0}_{B_2\times B_2} \end{bmatrix}, \\ \mathbf{H}_{ERR} &= \mathbf{H}_{ERR1} + \mathbf{H}_{ERR2}. \end{aligned}$$

Post equalization means multiplying the received data from left hand side by $\mathbf{H}_{APP}^{-1}$:

$$\begin{aligned} \hat{\mathbf{d}}_{\text{post}} &= \mathbf{H}_{APP}^{-1}\mathbf{x} \\ &= \underbrace{\mathbf{H}_{APP}^{-1}\mathbf{H}_{APP}}_{\mathbf{I}}\mathbf{d} + \underbrace{\mathbf{H}_{APP}^{-1}\mathbf{H}_{ERR}\mathbf{d}}_{\text{approximation error}} + \mathbf{H}_{APP}^{-1}\mathbf{n} \end{aligned}$$

If pre equalization is performed the data $\mathbf{d}$ is multiplied by $\mathbf{H}_{APP}^{-1}$ before it is transmitted over the channel:

$$\begin{aligned} \hat{\mathbf{d}}_{\text{pre}} &= \mathbf{H}\mathbf{H}_{APP}^{-1}\mathbf{d} + \mathbf{n} \\ &= \underbrace{\mathbf{H}_{APP}\mathbf{H}_{APP}^{-1}}_{\mathbf{I}}\mathbf{d} + \underbrace{\mathbf{H}_{ERR}\mathbf{H}_{APP}^{-1}\mathbf{d}}_{\text{approximation error}} + \mathbf{n} \qquad (4) \end{aligned}$$

The absolute value of the approximation error in the post equalization case

$$\mathbf{e}_{\text{post}} = \text{abs}\left(\mathbf{H}_{APP}^{-1}\mathbf{H}_{ERR}\mathbf{d}\right)$$

averaged over an ensemble of realizations (channel and data) is shaped like a bathtub if it is plotted as a function of the symbol index. By moving the blocks of the matrix $\mathbf{H}_{APP}^{-1}$ along the diagonal every data symbol can be estimated at every error level (overlapping method). In the case of pre equalization the absolute approximation error

$$\mathbf{e}_{\text{pre}} = \text{abs}\left(\mathbf{H}_{ERR}\mathbf{H}_{APP}^{-1}\mathbf{d}\right)$$

averaged over an ensemble of realizations (channel and data) is zero from the $(L)^{\text{th}}$ to the $(B_2)^{\text{th}}$ entry of one block and non zero in the remaining part of one block. During the symbol indices with nonzero approximation error we have either to introduce redundancy into the data or we can remove the error by suitable computations at the receiver. The approximation errors as function of the time index are plotted in figure 2. Bit error rates for the post equalization scenarios can be found in [8, 9].

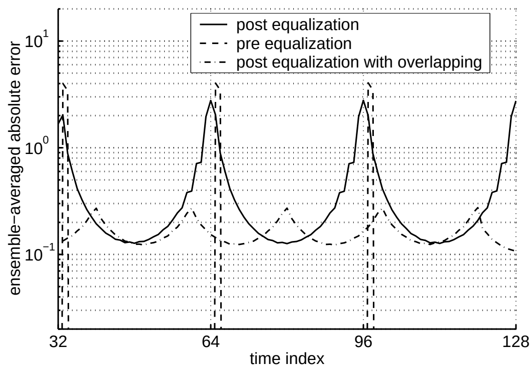

Figure 2: The approximation error in the case of "cyclic" post and pre equalization as a function of the time index is plotted. Additionally the effect of overlapping ($P = 6$, $Q = 6$) in the case of post equalization is shown. The Parameters are $L = 3$ (channel length), $B = 32$ (block size). The data consists of a Bernoulli sequence of the set $\{1, -1\}$. Each tap of the channel is equally distributed in the range of $[0, 1]$. The absolute estimation error is averaged over 50000 trails.

If the noise vector $\mathbf{n}$ in equation 4 is assumed to be zero, the estimated data $\hat{\mathbf{d}}_{pre}$ is equal to the true data $\mathbf{d}$ added by the approximation error, which is zero in the last $B_2 - L + 1$ elements of each block (see figure 3). If we assume, that the channel vector $\mathbf{h}$ is known at the receiver the error matrix $\mathbf{H}_{ERR}\mathbf{H}_{APP}^{-1}$ can

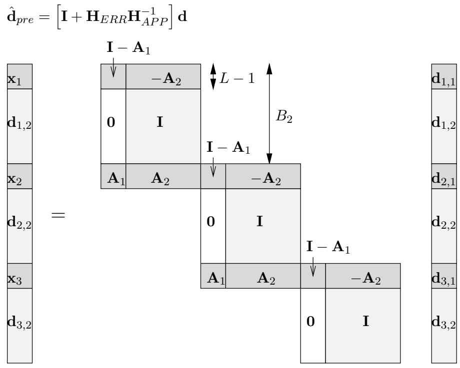

Figure 3: Structure of error matrix.

be computed. This will lead to an system of equations showing the structure depicted in figure 3. The light gray parts of the vectors $\hat{\mathbf{d}}_{pre}$ and $\mathbf{d}$ are identical. At the receiver $\hat{\mathbf{d}}_{pre}$ and the matrix $\mathbf{A} = [\mathbf{A}_1\ \mathbf{A}_2]$ are known (assumption that the the channel vector is known at the receiver). The vector part $\mathbf{d}_{1,1}$ can then be computed by first subtracting the second part of $-\mathbf{A}$ (called $\mathbf{A}_2$) multiplied by $\mathbf{d}_{1,2}$ from $\mathbf{x}_1$. Then solving the system of equation $\mathbf{x}_1 - \mathbf{A}_2\mathbf{d}_{1,2} = (\mathbf{I}_{L-1} + \mathbf{A}_1)\mathbf{d}_{1,1}$ renders the missing estimates of the transmitted symbols of the first data block. For it the matrix $\mathbf{I}_{L-1} + \mathbf{A}_1$ need to be non singular. This is usually the case. The data $\mathbf{d}_{2,1}$ can be computed by solving the system of equation $\mathbf{x}_2 - \mathbf{A}[\mathbf{d}_{1,1}; \mathbf{d}_{1,2}] + \mathbf{A}_2\mathbf{d}_{2,2} = (\mathbf{I}_{L-1} + \mathbf{A}_1)\mathbf{d}_{2,1}$. The matrix vector product $\mathbf{A}[\mathbf{d}_{1,1}; \mathbf{d}_{1,2}] = \mathbf{x}_1 - \mathbf{d}_{1,1}$, so that it need not be computed. The following blocks are processed in a similar fashion. In the noisy case the estimates of $\mathbf{d}_{1,1}$, $\mathbf{d}_{2,1}$ and $\mathbf{d}_{3,1}$ can suffer from severe noise enhancement. This problem can be circumvented if all received data blocks ($\mathbf{x}_2$ and $\mathbf{x}_3$) that contain information of one data block ($\mathbf{d}_{2,1}$) are taken into account for estimation. The piece of information from the following block ($\mathbf{d}_{3,1}$) is than assumed to be additional noise.

Next the overlapping method using Toeplitz submatrices is considered. The description is almost similar to the cyclic case with the exception that the Toeplitz submatrix is used instead of the circulant one. On contrast to the cyclic case the approximate inversion of the channel matrix can only be performed from the left side, which is equivalent to perform it at the receiver.

IV. Conclusions

The cyclic prefix method results in four possible transmission systems: OFDM and CP-SC, each with either pre or post equalization. The zero padding method result in the transmission system ZP-SC in which the pseudo inverse is multiplied to the received vector. However, ZP block transmissions can be transfered to the cyclic method, so that the four above mentioned transmission systems can be applied too.

Overlapping represents a method that approximately removes ISI in a block wise fashion. The processing stages of each block are similar to the ones in the CP-SC and ZP-SC transmission systems. It therefore offers the feasibility of detecting data with similar hardware, independently whether guard periods are transmitted or not. Overlapping techniques are also used in efficient solutions of the multi user detection problem [11] and in data detection of CDMA signals [6].

Post and pre equalization is feasible if circulant submatrices are selected. In the case of Toeplitz submatrices only post (or balanced) equalization is feasible. In the case of pre equalization by using circulant submatrices some of the received data symbols are erroneous if guard periods are not inserted. If the channel vector is known at the receiver, the errors can be corrected.

The three block wise equalization techniques overlapping with circulant submatrices, pre equalization with inverse circulant submatrices combined with error correction, and overlapping with Toeplitz submatrices can be used in an OFDM System to equalize the channel. Then the insertion of CPs can be omitted. They can be used in single carrier systems to remove the guard periods. This can be done by an FIR channel equalization filter as well. However, the overlapping methods use similar computations like CP-SC and ZP-SC, so that a multi mode device (transmissions with and without guard periods) with similar hardware is applicable.

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