On the Selection of a Per-Subchannel Equalizer for Time Dispersive SIMO Channels

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Abstract— **Filtered multitone modulation (FMT) represents a multicarrier transmission technique that shows significantly higher spectral efficiency than orthogonal frequency division multiplexing (OFDM). However, its complexity is higher. Among other things this results from the need of subchannel equalizers, which are more complex than the ones used in OFDM. Here, least square (LS) and minimum mean square error (MMSE) equalizers are considered. In a burst structured transmission scenario with multiple receiving antennas two systems of equations whose resolutions perform channel equalization of one subchannel can be established. One is block-cyclic (CP) structured, the other is block-Toeplitz (TP) structured. The system of equations are compared with each other in terms of their condition number, the bit error rate (BER), and the computational complexity. The solution of the block-cyclic system is very efficient. However, the block-Toeplitz system is better conditioned and results in a lower BER. The difference between the condition numbers of the systems decreases with the number of receiving antennas. The systems of equations resulting from the MMSE estimation are usually better conditioned than the ones resulting from the LS estimation.**

Index Terms—**least square, minimum mean square error, equalization, cyclic matrices, Toeplitz matrices, condition number, multiple receiving antennas, filtered multitone.**

I. Introduction

This article is concerned with the selection of a reasonable system of equations whose resolution performs channel equalization in broadband wireless communications. One system can be solved in a computational very efficient manner, however, it is sometimes ill-conditioned. The other is usually better conditioned but requires more computations. The resulting equalizers can for example be applied in single carrier transmission systems based on data blocking or as a per-subchannel equalizer in filtered multitone (FMT) systems.

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FMT is recently proposed for broadband transmissions over wireless channels [1]. Its functionality and efficient implementations are discussed in [3], [5], and [12]. FMT represents a multi carrier transmission concept that is different from OFDM. The major advantage of FMT over OFDM is significantly higher spectral efficiency. This results from the overhead produced by the cyclic prefix and the virtual carriers in OFDM systems which decrease the spectral efficiency in the wireless LAN standard Hiperlan/2 for example by $38,75\%$ [3], [6]. However, FMT systems are more complex to implement although efficient implementations based on the FFT and polyphase filtering are proposed [1], [3].

One main drawback that increases the complexity of FMT is the need of comparatively complex subchannel equalizers. In [1] and [5] decision feedback equalizers are suggested for FMT systems. In this paper we consider least square (LS) and minimum mean square error (MMSE) equalizers instead. Here, a scenario with multiple receiving antennas and a time dispersive nature of each subchannel is adopted. Modeling the scenario leads to a system of equations that can be either block-Toeplitz (TP) or block-cyclic (CP) structured. If the burst to be transmitted is structured into alternating sequences of training data and information data both systems of equations can be established. Then the systems are solved in the LS or MMSE sense. To assess the quality of the solution the condition number of the respective system matrix is selected. By means of the condition number the quality of the channel equalizers is assessed. The condition number changes with the selected system of equations and the number of applied receiving antennas. In the least square, one antenna case it turns out that the mean condition of the Toeplitz system is significantly better than the one in the cyclic system. By increasing the number of receiving antennas the condition of the two systems will con-

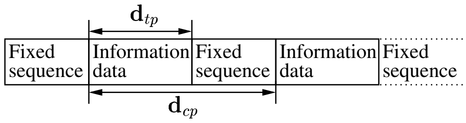

Fig. 1. Burst structure of a block processing system using fixed sequences as guard periods.

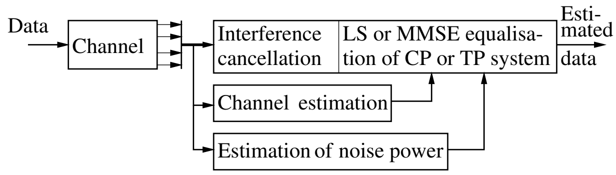

Fig. 2. Equalization of one channel in FMT systems.

verge against each other. Applying the MMSE criterion improves the condition of both systems. However, an estimation of the noise power becomes necessary. Beside the expected quality of the equalizer, that is described by the condition number, its computational complexity is determined by the selection of the respective system of equations. The cyclic system (CP) is solvable very efficiently. However, there exist some efficient algorithms and reasonable approximations to lower the complexity of solving the Toeplitz system (TP) considerably.

The article is organized as follows: First the data model and the different systems of equations (CP and TP) are introduced. Then the LS and MMSE equalization techniques in the one and multi antenna case are reviewed. In section 3 the ability to assess the different equalizers by the condition number of the respective system matrix is discussed. Histograms of the condition number for a time dispersive random channel, for the different equalizers, and for a various number of receiving antennas are computed in section 4. Implementation aspects of the receiver are considered in section 5. Finally, some conclusions are drawn.

II. Data Model

To enable efficient channel equalization in transmissions over severe time dispersive channels a block wise processing is often adopted [9], [10], [14]. To avoid interference between blocks following each other a guard period is usually inserted. It contains nulls, a cyclic prefix or a fixed sequence [2], [14]. The burst structure in the case of frequently inserting a fixed sequence as guard period is depicted in figure 1. The sequence can be used for channel estimation and synchronization. Furthermore it enables the cyclic deconvolution for channel equalization.

In a time discrete, synchronous transmission model the channel equalization problem can be denoted in a system of equations. Solving the system of equations is equivalent to performing channel equalization. Based on the burst structure depicted in figure 1 two different systems of equations can be established. If we exploit the fixed sequence as cyclic prefix the system of equations is cyclically structured:

$$\mathbf{x}_{cp} = \tilde{\mathbf{H}}\mathbf{d}_{cp} + \mathbf{n}_{cp},$$

i.e., the matrix $\tilde{\mathbf{H}} \in \mathbb{C}^{B_{cp} \times B_{cp}}$ describes cyclic convolution of the data sequence $\mathbf{d}_{cp}$ and the channel impulse response (CIR) $\mathbf{h} \in \mathbb{C}^{L}$. The received vector is denoted by $\mathbf{x}_{cp}$, the vector $\mathbf{n}_{cp}$ models noise. Since the fixed sequence is assumed to be known at the receiver and the CIR is already estimated the interference caused by the fixed sequence can be eliminated first. This leads to another system of equations:

$$\mathbf{x}_{tp} = \mathbf{H}_{tp}\mathbf{d}_{tp} + \mathbf{n}_{tp}.$$

The channel convolution matrix $\mathbf{H}_{tp} \in \mathbb{C}^{(B_{tp}+L-1) \times B_{tp}}$ is Toeplitz structured. On contrast to the cyclic system the Toeplitz system has in general full column rank independently of the actual CIR (exception CIR $= \mathbf{0}$).

Figure 2 shows the considered transmission scenario. It can be one subchannel in FMT based transmissions or a single carrier transmission system. The interference cancellation is only performed if the Toeplitz system is established. The noise power is only estimated if the MMSE criterion is selected. Subsequently a single input multiple output channel is considered. Thereby the structure of the considered systems of equations remain similar. The estimated channel impulse response between the transmitter and the m^{th} receiving antenna is given by

$$\mathbf{h}_m = \begin{bmatrix} h_{m,1} & h_{m,2} & \dots & h_{m,L} \end{bmatrix}^T \in \mathbb{C}^{L}, \quad 1 \leq m \leq M.$$

The M channel impulse responses can be merged in one vector

$$\mathbf{h} = \text{vec}\left\{ \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \dots & \mathbf{h}_M \end{bmatrix} \right\} \in \mathbb{C}^{ML}.$$

Thereby the vec operator is defined by

$$\text{vec}\left\{ \begin{bmatrix} a_1 & b_1 & \dots \\ a_2 & b_2 & \dots \end{bmatrix} \right\} = \begin{bmatrix} a_1 & b_1 & a_2 & b_2 & \dots \end{bmatrix}^T.$$

The usage of multiple receiving antennas can be incorporated in the cyclic model by using a block-cyclic system matrix $\tilde{\mathbf{H}}_{bc} \in \mathbb{C}^{MB_{cp} \times B_{cp}}$:

$$\mathbf{x}_{bcp} = \tilde{\mathbf{H}}_{bc}\mathbf{d}_{cp} + \mathbf{n}_{bcp}. \quad (1)$$

If the interference from training data is removed first the resulting system matrix $\mathbf{H}_{bt} \in \mathbb{C}^{M(B_{tp}+L-1) \times B_{tp}}$ is block-Toeplitz structured:

$$\mathbf{x}_{btp} = \mathbf{H}_{bt}\mathbf{d}_{tp} + \mathbf{n}_{btp}. \quad (2)$$

From equations 1 and 2 the receiver knows the channel and the received data, it does not know the transmitted data and the noise. Usually these systems of equations are not solvable, so that methods need to be identified that render an estimate of the data. This can be the LS or MMSE method. If the later is applied to the block systems the following equations are obtained:

$$\begin{aligned}(\tilde{\mathbf{H}}_{bc}^H\tilde{\mathbf{H}}_{bc} + \hat{\sigma}\mathbf{I})\hat{\mathbf{d}}_{cp,mmse} &= \tilde{\mathbf{H}}_{bc}^H\mathbf{x}_{bcp}, \quad (3)\\ (\mathbf{H}_{bt}^H\mathbf{H}_{bt} + \hat{\sigma}\mathbf{I})\hat{\mathbf{d}}_{tp,mmse} &= \mathbf{H}_{bt}^H\mathbf{x}_{btp}. \quad (4)\end{aligned}$$

If the estimation of the noise power $\hat{\sigma}^2$ is set to zero the MMSE solution is equivalent to the least square solution. Then the existence of $(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})^{-1}$ is not guaranteed. This is a major advantage of the MMSE over the LS method. Since $\hat{\sigma}$ is not zero in practical systems $(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} + \hat{\sigma}\mathbf{I})$ is always invertible. However, the noise power need to be estimated, which increases the complexity of the receiver. There are two possibilities to assure the invertibility. Either MMSE is selected instead of LS or the Toeplitz system is selected, whose system matrix is invertible in both the LS and MMSE estimation. Next the reasonable selection of the system is discussed. Criterions are the quality of the estimates and the complexity of computing the estimates. The quality can be measured with the condition number of the respective system matrix [7], [11]. In the following the connection between the condition number and the noise enhancement is discussed.

III. Condition Number and Noise Enhancement

The sensitivity of both the block-cyclic and the block-Toeplitz system to perturbations in the data is analyzed. It is assumed that the CIR is estimated exactly, which implies that the true channel convolution matrix is known. The received vector $\mathbf{r}$, which denotes either $\mathbf{x}_{btp}$ or $\mathbf{x}_{bcp}$, is obtained by multiplying the channel matrix $\mathbf{A}$, which denotes either $\mathbf{H}_{bt}$ or $\tilde{\mathbf{H}}_{bc}$, by the data $\mathbf{d}$, which denotes either $\mathbf{d}_{tp}$ or $\mathbf{d}_{cp}$, and by adding the noise $\mathbf{n}$, which denotes either $\mathbf{n}_{btp}$ or $\mathbf{n}_{bcp}$. Then we can express both equation 3 and 4 in the case of LS or MMSE estimation by

$$\mathbf{C}_{ls,mmse}\hat{\mathbf{d}}_{ls,mmse} = \mathbf{A}^H\mathbf{r}, \quad \text{where} \quad (5)$$
$$\mathbf{C}_{ls} = \mathbf{A}^H\mathbf{A} \quad \text{and} \quad \mathbf{C}_{mmse} = \mathbf{A}^H\mathbf{A} + \hat{\sigma}\mathbf{I}.$$

To assess the accuracy of the solution of the system a small perturbation of the right hand side $\delta(\mathbf{A}^H\mathbf{r})$ is assumed. The perturbed system is then given by:

$$\mathbf{C}_{ls,mmse}(\hat{\mathbf{d}}_{ls,mmse} + \delta(\hat{\mathbf{d}}_{ls,mmse})) = \mathbf{A}^H\mathbf{r} + \delta(\mathbf{A}^H\mathbf{r})$$

The error of the solution $\delta(\hat{\mathbf{d}}_{ls,mmse})$ is then determined by (if $\mathbf{C}_{ls}^{-1}$ exists):

$$\delta(\hat{\mathbf{d}}_{ls,mmse}) = \mathbf{C}_{ls,mmse}^{-1}\delta(\mathbf{A}^H\mathbf{r}). \quad (6)$$

By applying a law of matrix norms ($\|\mathbf{B}\mathbf{y}\| \leq \|\mathbf{B}\|\|\mathbf{y}\|$) to equations 5 and 6 we can verify that

$$\begin{aligned}\|\mathbf{A}^H\mathbf{r}\| &\leq \|\mathbf{C}_{ls,mmse}\|\|\hat{\mathbf{d}}_{ls,mmse}\| \quad \text{and}\\ \|\delta(\hat{\mathbf{d}}_{ls,mmse})\| &\leq \|\mathbf{C}_{ls,mmse}^{-1}\|\|\delta(\mathbf{A}^H\mathbf{r})\|.\end{aligned}$$

After multiplying the two inequations we receive

$$\frac{\|\delta(\hat{\mathbf{d}}_{ls,mmse})\|}{\|\hat{\mathbf{d}}_{ls,mmse}\|} \leq \|\mathbf{C}_{ls,mmse}^{-1}\|\|\mathbf{C}_{ls,mmse}\|\frac{\|\delta(\mathbf{A}^H\mathbf{r})\|}{\|\mathbf{A}^H\mathbf{r}\|}.$$

The ratio of the solution error norm to the true solution norm is bounded by the ratio of the norm of the right hand side perturbation to the norm of the true right hand side multiplied by the factor $\|\mathbf{C}_{ls,mmse}^{-1}\|\|\mathbf{C}_{ls,mmse}\|$. This is the condition number of $\mathbf{C}_{ls,mmse}$. The condition number measures the sensitivity of the solution of equation 5 to perturbations in the data on the right hand side. The perturbation results from a limited accuracy in a digital processing unit and from deviations of the channel estimates from the true values. A large condition indicates a large sensitivity of the solution to perturbations whereas a low condition number indicates a low sensitivity. The condition number depends on the matrix norm that is selected. Here the two norm is selected. That is the ratio of the largest to the smallest singular value of the considered matrix.

However, the question how the condition number $\|\mathbf{C}_{ls,mmse}^{-1}\|\|\mathbf{C}_{ls,mmse}\|$ of the system matrix can describe the deviation between the estimated data $\hat{\mathbf{d}}_{ls,mmse}$ and the true data $\mathbf{d}$ is not answered yet. If the least square case of equation 5 is multiplied by $\mathbf{C}_{ls}^{-1}$ (existence of this inverse is not guaranteed) an equation that contains a connection between the LS estimated data and the true data is obtained:

$$\hat{\mathbf{d}}_{ls} = \mathbf{C}_{ls}^{-1}\mathbf{A}^H\mathbf{r} = \mathbf{d} + \mathbf{C}_{ls}^{-1}\mathbf{A}^H\mathbf{n}.$$

The LS estimates $\hat{\mathbf{d}}_{ls}$ are composed of the true data $\mathbf{d}$ and the noise $\mathbf{A}^H\mathbf{n}$ that is enhanced by $\mathbf{C}_{ls}^{-1}$. The enhanced noise is the solution of the system $\mathbf{C}_{ls}\mathbf{y} = \mathbf{0} + \mathbf{A}^H\mathbf{n}$. The noise $\mathbf{A}^H\mathbf{n}$ can be considered as the perturbation of the right hand side. Then the condition number $\|\mathbf{C}_{ls}^{-1}\|\|\mathbf{C}_{ls}\|$ measures the enhancement of noise and thus gives an indication of the deviation between the estimated data $\hat{\mathbf{d}}_{ls}$ and the true data $\mathbf{d}$. Therefore we expect a strong deviation between the estimates $\hat{\mathbf{d}}_{ls}$ and the true data $\mathbf{d}$ if the condition number of the matrix $\mathbf{C}_{ls}$ is large. If the MMSE case of equation 5 is multiplied by $\mathbf{C}_{mmse}^{-1}$ (existence is guaranteed if $\hat{\sigma} \neq 0$) the following equation is obtained:

$$\hat{\mathbf{d}}_{mmse} = \mathbf{C}_{mmse}^{-1}\mathbf{A}^H\mathbf{r} = \mathbf{C}_{mmse}^{-1}\mathbf{A}^H\mathbf{A}\mathbf{d}+\mathbf{C}_{mmse}^{-1}\mathbf{A}^H\mathbf{n}.$$

The MMSE estimates $\hat{\mathbf{d}}_{mmse}$ are equivalent to the true data $\mathbf{d}$ only if $\hat{\sigma} = 0$ and $\mathbf{n} = \mathbf{0}$ (and $\mathbf{C}_{mmse}^{-1}$ exists). If $\hat{\sigma}$ is small ($\mathbf{C}_{mmse}^{-1}\mathbf{A}^H\mathbf{A} \approx \mathbf{I}$), then the deviation from the true data is mainly determined by $\mathbf{C}_{mmse}^{-1}\mathbf{A}^H\mathbf{n}$. Then the noise $\mathbf{A}^H\mathbf{n}$ is strongly enhanced if the condition number $\|\mathbf{C}_{mmse}^{-1}\|\|\mathbf{C}_{mmse}\|$ is large. For larger values of $\hat{\sigma}$ the condition number will decrease (lower noise enhancement). However, the deviation between $\mathbf{C}_{mmse}^{-1}\mathbf{A}^H\mathbf{A}$ and the identity matrix $\mathbf{I}$ becomes larger, which increases the deviation of the estimated data $\hat{\mathbf{d}}_{mmse}$ from the true data $\mathbf{d}$. In the next section histograms of the condition numbers are presented for different parameter sets.

IV. Simulations

The condition of the systems is measured for an L-tap channel ($L = 8$) in dependence of the selected equation system (CP or TP), the number of receiving antennas (1, 2 or 4) and the estimation technique (LS or MMSE). The block size of the cyclic system is set to $B_{cy} = 64$ and of the Toeplitz system to $B_{tp} = 57$, respectively. The taps of the CIR are complex random numbers. The real and imaginary part are normally distributed with zero mean and unit variance. In figure 3 the histogram of the condition numbers of the CP and TP systems by using least square equalization are displayed for one, two and four receiving antennas. In the one antenna case the condition of the LS Toeplitz system is significantly better than the one in the LS cyclic system. By increasing the number of receiving antennas the difference of the histograms becomes smaller. Furthermore the usage of multiple receiving antennas decreases the condition numbers of both systems. Applying the MMSE criterion improves the condition of all systems. In figure 4 the mean condition of the CP and TP systems as a function of the square root of the noise power $\hat{\sigma}$ is illustrated. A large $\hat{\sigma}$ results in a small condition and in almost no difference of the condition of the CP and TP systems whereas for a small $\hat{\sigma}$ the mean condition of the TP system is smaller than for the CP system.

In [9] the better conditioned system shows a significantly better BER. Since an ill-conditioned system can result in severe noise enhancement it is expected that it effects the BER of the transmission system applying the described equalizers. In figure 5 the BER as function of the SNR in the one antenna case for the CP and TP systems are shown (BPSK modulation). Here the LS and MMSE

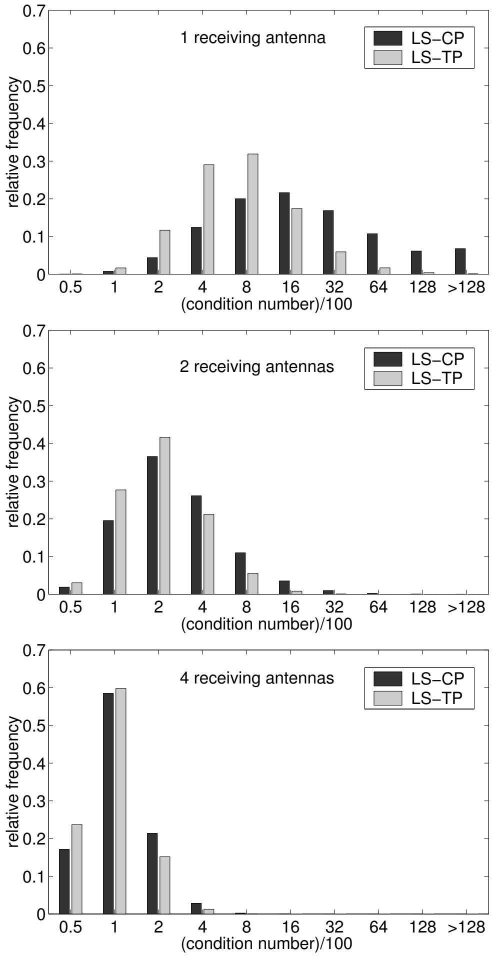

Fig. 3. Histogram of condition numbers of the CP and TP system matrices for a varying number of receiving antennas by applying least square estimation.

equalization is applied. The TP method renders lower bit error rates than the CP method for both the LS and MMSE equalization. For a low noise level MMSE and LS should render similar bit error rates. However, the system matrix is sometimes ill-conditioned, so that the noise level in the case of 21 dB is still too large for the techniques to converge against each other. Next it is focused on the efficient implementation for solving the systems of equations.

V. Efficient Implementation

The reason why block-cyclic systems are often preferred is the existence of efficient methods of resolution. Thereby the eigenvalue decomposition of cyclic matrices is exploited. The matrix $\mathbf{C} = \tilde{\mathbf{H}}_{bc}^H\tilde{\mathbf{H}}_{bc} + \hat{\sigma}\mathbf{I} \in \mathbb{C}^{B_{cy}\times B_{cy}}$ is cyclic and M-fold smaller than $\tilde{\mathbf{H}}_{bc}$. The eigenvalue decomposition is given by $\mathbf{C} = \mathbf{F}^{-1}\mathbf{\Lambda}\mathbf{F}$. The DFT matrix $\mathbf{F}$ can be implemented by using the FFT. The diagonal matrix $\mathbf{\Lambda}$ can be computed by the FFT from the first column of $\mathbf{C}$: $\mathbf{\Lambda} = diag(\mathbf{F}(\mathbf{C}(:,1)))$. The solution of the block-cyclic system requires the computation of $\mathbf{C}$, three FFTs

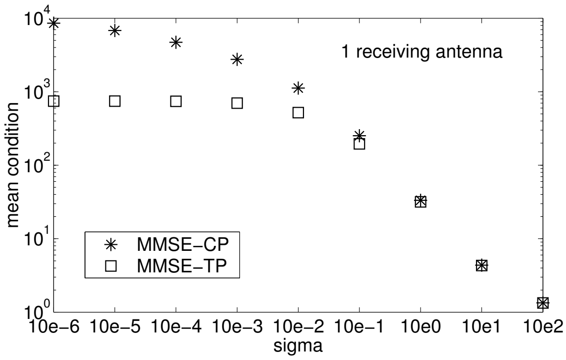

Fig. 4. Mean condition numbers for MMSE estimation as a function of sigma for different equation systems (TP, CP).

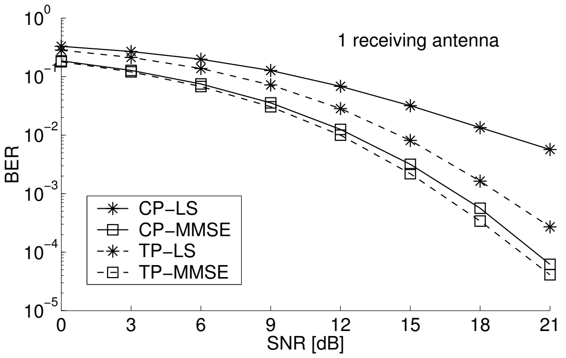

Fig. 5. Bit error rate of BPSK transmission with LS or MMSE equalization for different systems of equation (CP or TP).

of size B_{cp}, and one division per estimated data symbol. However, there are also efficient techniques to solve the block-Toeplitz system, e.g., the approximated Cholesky factorization [13], the method of cyclic reduction [4], and the Schur algorithm [8].

VI. Conclusion

Least square and minimum mean square error equalizers for time dispersive random channels were considered. If the transmitted burst is structured into alternating sequences of data and training two systems of equations that describe the transmission can be established. One is block-cyclic, one is block-Toeplitz structured. This holds for one and multiple receiving antennas. Solving the systems in the LS or MMSE sense performs channel equalization. The solution of the block-cyclic one is computational very efficient if FFT computations are applied. However, the condition number of the cyclic system is often larger than the one of the Toeplitz systems. This results in larger noise enhancement and thus in a higher bit error rate of the corresponding transmission system. The condition number of the correlation matrix of the respective system can give an indication of the quality of the estimated data. Therefore comparing the condition number of the respective correlation matrix is a coarse method that gives information about the BER to be expected. The use of multiple receiving antennas will lower the condition number in both systems. The larger the number of receiving antennas the smaller is the difference of the histogram of the condition number of the CP and TP system. The MMSE equation is usually better conditioned than the LS equation. If many receiving antennas are applied the CP system has lower complexity and similar quality as the TP system. If only a few antennas are applied, the TP system is significantly better conditioned, however, its solution requires more computations.

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