EFFICIENT DATA DETECTION ALGORITHMS IN SINGLE- AND MULTI-CARRIER SYSTEMS WITHOUT THE NECESSITY OF A GUARD PERIOD

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ABSTRACT

To enable efficient detection strategies in wireless communications a guard period is frequently inserted into the transmitted data symbols. This is the case in OFDM systems and in single-carrier systems with frequency domain equalization. Especially for transmissions over wireless channels with long delay spreads the required guard period is quite long. Here we present efficient data detection algorithms that are applicable if a guard period is not inserted after every symbol or if it is omitted completely. The increase of the computational requirements in the base station is moderate and can be implemented via parallel processing. Finally, an OFDM system without guard periods is presented as an example.

1. INTRODUCTION

This work focuses on transmission systems proposed for high data rate transmissions over wireless channels without the necessity of guard periods. The insertion of a cyclic prefix (CP) in single-carrier systems with frequency domain equalization and in CP-OFDM systems decreases the information data rate significantly. In the wireless LAN standards HiperLan/2 or IEEE802.11a about 20 percent of a data burst consists of the cyclic prefix, which obviously contains no information data [1]. In addition to the decreased information data rate the transmit power per datum is increased by 20 percent if a CP is transmitted instead of information data. Therefore, it is desirable to avoid the CP.

To enable efficient detection strategies based on FFT computations a guard period that consists of either a cyclic prefix or zero pads (ZP) is frequently inserted into the transmitted data symbols in both single- and multi-carrier systems. Therefore the information data rate is decreased. Here we present efficient data detection algorithms that are applicable if a guard period is not inserted after every symbol or if it is omitted completely. Thereby only a moderate increase of the computational requirements becomes necessary. An approximation technique is given that lowers the computational requirements and enables parallel processing. The technique is applicable to multiple receive antennas, which improve the BER performance without increasing computational demands significantly.

In section 2 a data model for the known CP-OFDM and single-carrier system with CP is derived before it is extended to systems omitting the CP. After describing the system model in section 2 the approximated data detection technique is presented in section 3. The technique is applied to an OFDM system without CP and is assessed by simulations in section 4. Finally some conclusions are drawn.

2. SYSTEM MODEL

In the multi-carrier case the data symbols of one block are modulated on different subcarriers by using the IFFT. Then either a cyclic prefix or zero pads are inserted to combat the problems of inter symbol and inter carrier interference caused by the wireless channel. The guard period must be at least as long as the maximum delay spread of the channel. After passing through the mobile vector channel the interfering parts of subsequent OFDM symbols are discarded. The transmitted data symbols can be estimated at the receiver by performing an FFT and a division by the value of the channel transfer function at the respective subcarrier frequency. Then the antenna output can be combined properly. This is a computational very cheap possibility to perform data detection.

If the IFFT is moved to the receiver the overall complexity is retained. This system is known as single-carrier system with frequency domain equalization. To avoid noise amplification it is reasonable to apply the MMSE criteria for equalization. In fact both systems are similar in terms of BER performance and computational requirements [2, 3]. A difference is the possibility of adaptive modulation in OFDM systems as every subcarrier can be power and bit loaded separately according to the respective SNR. The advantage of adaptive modulation is decreased if multiple receive antennas are utilized. However, adaptive modulation

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in single-carrier systems is possible in the time domain. The single-carrier technique is especially suited for an uplink scenario as the signal processing becomes computational very cheap in the transmitter and the amplifier stage does not have to deal with the peak-to-average power ratio (PAR) problem.

Based on the transmission system modeled in figure 1 a mathematical description is derived. Each symbol block

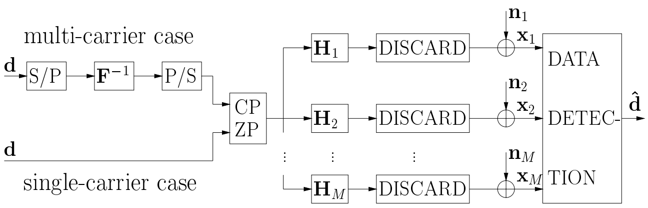


Fig. 1. Transmission system with multiple receiving antennas.

contains J data symbols. Only one data block is considered as it can be processed independently due to the guard period:

$$\mathbf{d} \;=\; \begin{bmatrix} d_1 & d_2 & \dots & d_J \end{bmatrix} \in \mathbb{C}^J.$$

A block of data symbols $\mathbf{d}$ is multiplied by a DFT matrix to perform subcarrier modulation:

$$\mathbf{F}^{-1} \;=\; \sqrt{J} \cdot \text{ifft}(\mathbf{I}_J) \in \mathbb{C}^{J \times J}.$$

The matrix $\mathbf{I}_J$ is the identity matrix of size J and ifft denotes a MATLAB-like function to calculate the IDFT. The modulated data is cyclically extended and then passed through a mobile vector channel, which is assumed to be time invariant during one data block such that it can be described by its channel impulse responses $\mathbf{h}_m$ of length W. The impulse responses for all M antennas can be merged into one:

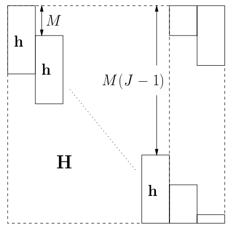
$$\begin{aligned} \mathbf{h}_m &= \begin{bmatrix} h_{m,1} & h_{m,2} & \dots & h_{m,W} \end{bmatrix}^T \in \mathbb{C}^W, \\ & \quad 1 \le m \le M, \\ \mathbf{h} &= \text{vec}\left\{ \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \dots & \mathbf{h}_M \end{bmatrix} \right\} \in \mathbb{C}^{MW}. \end{aligned}$$

Here, the vec operator is defined by

$$\text{vec}\left\{ \begin{bmatrix} a_1 & a_2 & \dots \\ b_1 & b_2 & \dots \end{bmatrix} \right\} = \begin{bmatrix} a_1 & b_1 & a_2 & b_2 & \dots \end{bmatrix}^T.$$

The convolution matrix $\mathbf{H} \in \mathbb{C}^{M(J+W-1)\times J}$ referring to one OFDM symbol retains its structure, namely band-structured block-Toeplitz, even for multiple receiving antennas. Combining the ZP/CP, the channels, and the discard stages results in a block-cyclic matrix:

$\tilde{\mathbf{H}} =$



$\in \mathbb{C}^{MJ \times J}.$

If zero pads are inserted the BER performance can be improved at the expense of higher computational requirements. However, the ZP case can be reduced to the CP case by adding the last $W-1$ samples to the first. The differences between the usage of either a CP or ZP is investigated in [4]. The received vector at the m-th antenna and the received vector at the data detection stage, respectively, are arranged as follows:

$$\begin{aligned} \mathbf{x}_m &= \begin{bmatrix} x_{m,1} & x_{m,2} & \dots & x_{m,J} \end{bmatrix}^T \in \mathbb{C}^J, \\ \mathbf{x} &= \text{vec}\left\{ \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_M \end{bmatrix} \right\} \in \mathbb{C}^{MJ}. \end{aligned}$$

The received vector can be expressed in a system of equations:

$$\mathbf{x} = \tilde{\mathbf{H}}\mathbf{F}^{-1}\mathbf{d} + \mathbf{n} \in \mathbb{C}^{MJ}.$$

In a single-carrier system the subcarrier modulation is omitted:

$$\mathbf{x} = \tilde{\mathbf{H}}\mathbf{d} + \mathbf{n} \in \mathbb{C}^{MJ}.$$

The noise vector $\mathbf{n}$ is arranged in a similar fashion as $\mathbf{x}$. To detect the data either the cyclic channel matrix $\tilde{\mathbf{H}}$ in single-carrier or the matrix $\tilde{\mathbf{H}} \cdot \mathbf{F}^{-1}$ in multi-carrier systems needs to be inverted. The efficient equalization in the frequency domain in both systems is based on the fact that the columns of the IDFT matrix, the subcarriers, are the eigenvectors of any cyclic matrix. This becomes obvious at the eigenvalue decomposition (EVD) of the m-th cyclically extended channel matrix:

$$\tilde{\mathbf{H}}_m = \mathbf{F}^{-1}\mathbf{D}\mathbf{F}.$$

The size of the cyclic channel matrix defines the size of the DFTs necessary to invert the matrices. The size of the cyclic channel matrix is defined by the distance between two subsequent CPs (J) in both single- and multi-carrier systems. The diagonal matrix $\mathbf{D}$ contains the eigenvalues on its diagonal, which are calculated by the DFT of the m-th channel impulse response zero padded to the right size:

$$\text{diag}(\mathbf{D}) = \mathbf{F}\mathbf{H}_m(:,1),$$

where $\text{diag}(\mathbf{D})$ denotes the diagonal elements of $\mathbf{D}$ rearranged as a vector and $\mathbf{H}_m(:,1)$ is the first column of $\mathbf{H}_m$.

If the distance between two CPs is increased, the information rate is increased as well. The price is higher computational complexity as the involved FFTs grow with the increasing distance between two subsequent CPs. Therefore there is a trade off between information data rate and computational requirements. However, the computational requirements for performing channel equalization in the frequency domain with comparatively large FFT's can be lowered using the approximation technique given in the next chapter.

3. APPROXIMATING FREQUENCY DOMAIN EQUALIZATION

Using the eigenvalue decomposition, the block-cyclic channel matrix can be decomposed as follows:

$$\begin{aligned} \tilde{\mathbf{H}} &= \mathbf{F}_M^{-1}\mathbf{D}_{M,1}\mathbf{F}, \\ \text{where} \quad \mathbf{F}_M^{-1} &= \mathbf{F}^{-1} \otimes \mathbf{I}_M. \end{aligned}$$

The block-diagonal matrix $\mathbf{D}_{M,1}$ has the block size $M \times 1$. The Kronecker product is denoted by $\otimes$. The zero forcing (ZF) and minimum mean square error (MMSE) estimates are calculated as follows:

$$\begin{aligned} \text{ZF:} \quad \hat{\mathbf{d}} &= (\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})^{-1}\tilde{\mathbf{H}}^{\mathbf{H}}\mathbf{x} \\ &= \mathbf{F}^{-1}(\mathbf{D}_{M,1}^H\mathbf{D}_{M,1})^{-1}\mathbf{D}_{M,1}^H\mathbf{F}_M\mathbf{x}, \\ \text{MMSE:} \quad \hat{\mathbf{d}} &= (\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} + \sigma\mathbf{I})^{-1}\tilde{\mathbf{H}}^H\mathbf{x} \\ &= \mathbf{F}^{-1}(\mathbf{D}_{M,1}^H\mathbf{D}_{M,1} + \sigma\mathbf{I})^{-1}\mathbf{D}_{M,1}^H\mathbf{F}_M\mathbf{x}, \\ & \quad \sigma\mathbf{I} = \mathrm{E}\{\mathbf{n}\cdot\mathbf{n}^H\}. \end{aligned}$$

The DFT $\mathbf{F}_M$ can be performed in parallel by applying $\mathbf{F}$ to the received data symbols at each antenna. $\mathbf{F}$ can be implemented using the FFT. The computational complexity of this approach increases linearly with the number of antennas as for each additional antenna only one additional FFT needs to be applied. Because of the parallelism, the approximated frequency domain equalization is explained for the one antenna case. The idea is to divide the comparatively large channel matrix into smaller matrices which are to be cyclically extended and processed independently of each other. Therefore the size of the involved FFTs decreases. For example, instead of using one 512-FFT four 128-FFT can be used, which can be calculated in parallel. The algebraic accuracy is improved if the considered submatrices overlap with each other [5, 6]. The idea of the overlapping technique is illustrated in figure 2. This technique approximates exact frequency domain equalization so that the necessity of a guard period becomes dispensable. This technique is reasonably applicable in transmission systems in which a significant part of the transmitted data consists of guard periods. The performance of the technique is illustrated in the next section in which an OFDM system without CP is presented.

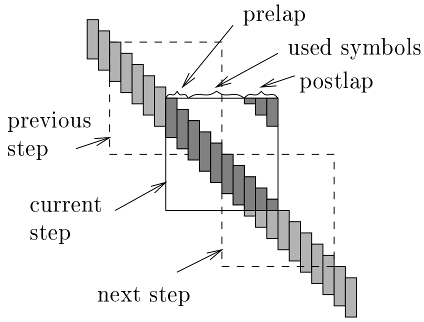


Fig. 2. Approximated frequency domain equalization.

4. SIMULATION AND EVALUATION

Here, a ZP is not transmitted after every OFDM symbol but once in a while, e.g., after each seventh OFDM symbol. Each block of OFDM symbols until a ZP is reached can be considered independently as subsequent blocks do not interfere with each other due to the ZP. As a ZP is not inserted after every OFDM symbol inter symbol and inter carrier interference occur. This interference needs to be eliminated first. This can be done efficiently using the presented method either by inverting the channel matrix exactly or by using the given approximation techniques. The system is depicted in figure 3. In the sequel, $\mathbf{F}_{symbol}$ refers to one OFDM symbol, whereas $\mathbf{F}_{block}$ refers to one block of OFDM symbols. The blocksize is given by the number of OFDM symbols without guard period between two subsequent ZPs.

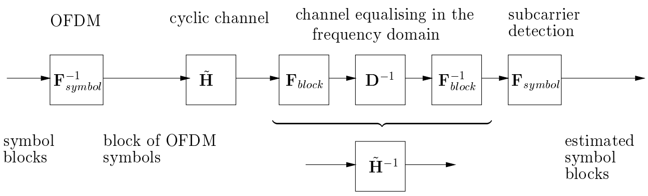


Fig. 3. OFDM system without guard period.

For example, if one OFDM symbol consists of $J = 64$ subcarriers ($\mathbf{F}_{symbol}$ corresponds to a 64-FFT) and $B = 7$ blocks are sent before a ZP is inserted and the channel is of length W=16, a 512-FFT ($\mathbf{F}_{block}$ corresponds to a 512-FFT) is needed to diagonalize the channel matrix with the radix-2 algorithm. The FFT matrices $\mathbf{F}_{block}$ and $\mathbf{F}_{symbol}$ are of the same size if a CP is inserted after each OFDM symbol. In this case the system is reduced to the CP-OFDM system as $\mathbf{F}_{block}^{-1}\mathbf{F}_{symbol} = \mathbf{I}$. In comparison to CP-OFDM the system without CP needs to spend two further comparatively large DFTs $\mathbf{F}_{block}$ and $\mathbf{F}_{block}^{-1}$ and a computational more expensive calculation of the diagonal matrix $\mathbf{D}^{-1}$.

In figure 4 the BER is plotted as a function of the SNR. The known CP-OFDM system is viewed as the reference system which has the lowest BER. For exact equalization in the frequency domain a 512-FFT is necessary. Here, the ZF method is chosen. Due to noise amplification the BER of this technique is slightly higher than that of CP-OFDM. But computational requirements will increase as shown in figure 5. Here, only the transmission system in figure 3 is considered. The influence of coding is not taken into account. The computational requirements can be reduced by the approximation technique presented. But reducing computational requirements results in a lower BER. Here smaller FFT sizes 256, 128 and 64 are applied. The quality of the approximation depends more distinctly on the size of the FFTs than on the prelap and postlap. The parameters in the legend of figure 4 are ”FFT-size - prelap - postlap”.

With the exact equalization technique (512-0-0) a similar

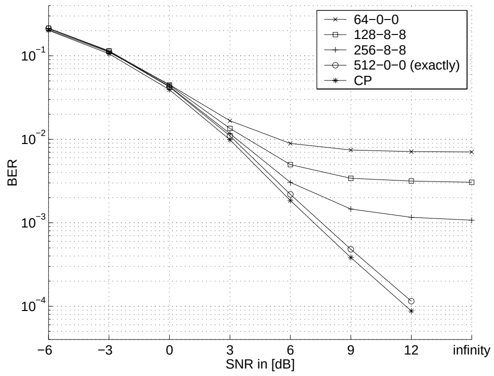


Fig. 4. BER of different transceiver techniques.

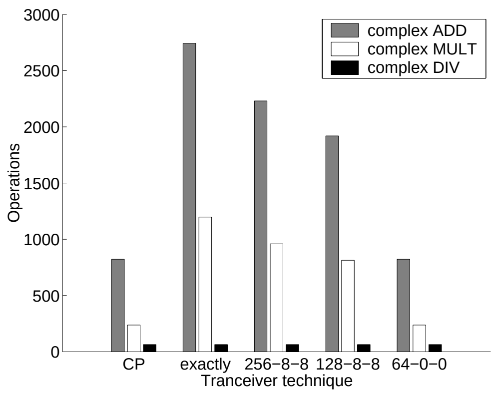


Fig. 5. Computational requirements of different transceiver techniques.

BER as in the CP-OFDM system can be achieved. At the same time the information data rate is increased by nearly 20 percent, but the computational demands of the receiver increases. The increase is moderate, see figure 5, so that this technique is investigated in the multiple antenna case. Simulations are shown for 1, 2, 4 and 8 antennas in figure 6.

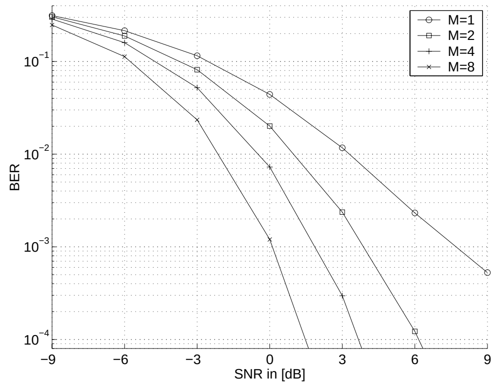


Fig. 6. BER of OFDM systems without CP applying antenna diversity.

5. CONCLUSIONS

Due to the neglect of the CP the information data rate is increased, but interference is caused in the received signal in both single and multicarrier systems. Here a computational efficient technique to cancel the interference was presented. The technique is efficiently applicable in the one- and multi-antenna case. The algorithms are inherently parallel and thus suited for a parallel implementation. The technique is especially suited for an uplink scenario as the additional computational requirements that become necessary due to the neglect of the CP can be performed at the receiver (base station), where it is not problematic.

6. REFERENCES

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