Hard Decision Based Low SNR Early Termination for LTE Turbo Decoding

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***Abstract*—In this paper a syndrome based Low SNR early termination (Low SNR ET) scheme for Turbo decoding is presented. The scheme is based only on hard decision binary computations and is therefore easy to realize. While Low SNR ET is a general principle, which can be applied in various scenarios, the focus of this work is on its application in the Long Term Evolution (LTE) system. It is shown that Low SNR ET is in particular very effective for typical LTE scenarios and that the porposed scheme can reduce the decoding iterations by about one third.**

I. Introduction

The data transmission in the Long Term Evolution (LTE) [1] employs Turbo coding [2] for error protection. Turbo codes feature an excellent bit error rate (BER) performance, compared to classical codes like convolutional or block codes. In case of ideal uncorrelated Gaussian inputs and very large code block lengths, Turbo codes can achieve close-to-capacity BERs. However, good performance has to be paid for with high computational complexity: In the case of LTE, Turbo codes are decoded iteratively by using two constituent decoders, which iteratively exchange their beliefs about the transmitted information bits in the form of Log Likelihood Ratios (LLRs). Typically the Max Log MAP algorithm [3] is used in the constituent decoders to generate these LLRs.

To reduce the computational complexity of the Turbo decoder, the number of iterations of the decoding process should be kept to a minimum. Ideally, considering the received code blocks this could be achieved by distinguishing between *decodable blocks*, which can be decoded (meaning all errors can be corrected by the Turbo decoder), and *undecodable blocks*, which cannot be decoded successfully. The minimum number of iterations would be achieved, if no iterations are done for undecodable blocks, and if just as many iterations as required are done for decodable blocks to correct all errors. Stopping the iteration process in case of undecodable blocks is called *Low SNR early termination (Low SNR ET)*. Stopping the iterations when all errors are corrected is called *High SNR early termination (High SNR ET)*.

Fig. 1 illustrates the different termination schemes. In all cases the black curve shows the average number of iterations, that is required for successful decoding at a certain SNR. Naturally the higher the SNR, the fewer iterations are required and vice versa. There is also a lower SNR bound, under which a successful decoding is not possible anymore. It should be noted here, that this is an idealized lower bound – in practice not only the SNR changes during system operation, but also other system parameters like code rate, block length, channel conditions (e.g. fading), that influence this lower bound. So it is not possible in practice to just use the current SNR estimate as a termination criterion.

The simplest termination approach, shown in the left-most part in Fig. 1, is to set the number of iterations of the Turbo decoder to some fixed value $i_{\max}$. The choice of $i_{\max}$ depends on various factors, like code, block length, expected SNR range, and is a trade-off between achievable BER and computational complexity. A typical value would be for example $i_{\max} = 8$. The obvious disadvantage of this approach is, that unneeded iterations are carried out for high SNR, if decoding is already successful after $i < i_{\max}$ iterations, and for low SNR, if the code block is still erroneous after $i = i_{\max}$ iterations.

The first problem can be avoided by using High SNR ET, which is illustrated in the middle of Fig. 1. In this case, the number of iterations is also set to a maximum value $i_{\max}$, but the iteration process is stopped as soon as the code block is error-free. In general, perfect High SNR ET cannot be realized, because it is impossible for the decoder to determine if a code block is error-free. However, several criteria have been proposed [4]–[8] to approximate High SNR ET by analyzing for example the LLR distribution of the extrinsic values or the number of sign changes in the extrinsic values.

In case of LTE, however, High SNR ET can be realized almost perfectly, because each code block has a 24 bit cyclic redundancy check (CRC24) attached to it. High SNR ET is then done easily by evaluating the CRC24 after each iteration and stopping the iteration process in case the CRC24 indicates that no errors are left. The probability of a wrong decision has

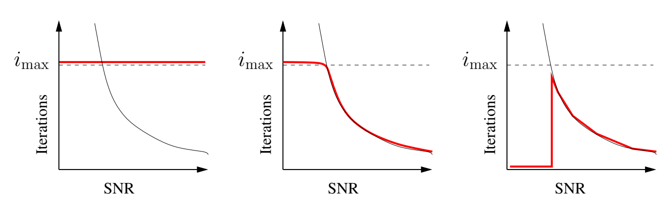


Fig. 1. Termination schemes for Turbo decoding, from left to right: Maximum Iteration Termination, High SNR ET, Combined Low & High SNR ET.

been shown to be negligible [9].

An optimal ET scheme would be a combination of High SNR ET and Low SNR ET, as shown in the right-most part of Fig. 1. In this case, the decoding effort in terms of iterations would be kept to a minimum. Realizing Low SNR ET is not straightforward in practice, because there is no definite way for the decoder to determine if a received code block is decodable in $i_{\max}$ iterations without actually decoding it. Nevertheless it is possible to approximate Low SNR ET by using certain criteria based on the constituent decoders outputs: For example, in [10], [11] the mean of the extrinsic LLR values is analyzed in order to detect undecodable blocks.

In this paper, we focus on Low SNR ET and its efficiency in typical LTE scenarios. A new syndrome based hard decision criterion for Low SNR ET is described and it is shown that, while its computational complexity is smaller, it can outperform a soft decision based criterion. It will also become clear that a good realization of Low SNR ET offers high potential for reduction of computational effort, especially in typical LTE scenarios.

This paper is organized as follows. In the following Sec. II the characteristics of the Turbo decoding process in a typical LTE scenario are discussed. After that, the proposed Low SNR ET scheme and possible criteria are described in Sec. III-A. Simulation results and analysis are presented in III-B. Conclusions are drawn in Sec. IV.

II. Turbo Decoding Characteristics in LTE

In order to promote the efficiency of a good realization of Low SNR ET in the LTE system, the characteristics of the Turbo decoder are investigated in a typical LTE scenario. This investigation is done by analyzing the number of Turbo decoder iterations at the working point. For the LTE scenario this working point is commonly assumed to be a target Block Error Rate (BLER) of 10%, which means that one out of ten received blocks is not decodable, such that the receiver has to issue an automatic repeat request (ARQ) for these blocks.

For the simulations presented in the remainder of this paper, the modulation and coding scheme (MCS) is set according to the Channel Quality Indicator (CQI) [1]. The simulations are carried out for the Typical Urban (TU) channel model, 1.4Mhz (CQI13) and 10MHz (CQI3 and CQI7) bandwidth and a receiver using receive diversity (1 TX, 2 RX). The carrier frequency is $f_C = 2.1$GHz and the maximum Doppler frequency is $f_{D,\max} = 19$Hz. The decoder uses the Max Log MAP algorithm and is set to a maximum iteration number of $i_{\max} = 8$, which is a typical choice. CRC24 is employed for High SNR ET. The analysis is carried out for CQI3 (QPSK, rate 193/1024, average code block length 3016), CQI7 (16 QAM, 378/1024, 5904), and CQI13 (64 QAM, 772/1024, 4344). It is assumed that no control channel information is transmitted by the eNodeB, meaning all resources are used for payload data only. The simulation environment is based on the LTE Simulator provided by [12].

Fig. 2 shows the complexity of the Turbo decoder in terms of executed iterations at the SNR working points corresponding to a BLER of 10% for the different CQIs. Fig. 2(a) on the left shows the probability $p(i)$ that an iteration i is executed. For each considered CQI this probability is shown for the case that High SNR ET is employed (CRC24 based) and for the case that High SNR and perfect Low SNR ET are combined. The latter is referred to as *Optimal ET*, as it assumes that for undecodable blocks no iterations are executed. Therefore, this serves as a lower bound for an ET scheme. Note that Fig. 2(a) is an idealized figure, because it is assumed that iterations can be stopped even before the first iteration, if the considered block is either error-free (High SNR ET) or undecodable (Low SNR ET).

For High SNR ET it can be noticed that the probability of the first iteration $p(1)$ is approximately one, i.e. that the number of error-free received blocks is very small, and that the probability $p(8) \approx 0.1$, i.e. that one out of ten blocks cannot be decoded successfully. The reason for the latter is of course the BLER working point of 10%. For the Optimal ET scheme, the probability $p(1)$ is about 0.9, while $p(8) \approx 0$. So it can easily be seen, that the Optimal ET curve is just the High SNR ET curve shifted by the offset of 0.1.

A more important conclusion from Fig. 2 is that the number of iterations in case of *successful* decoding is typically quite small. This can be seen from the dashed curves in Fig. 2(a). For example, for all considered CQIs, the probability, that the fourth iteration is executed is already below 0.1, and the average number of iterations is only about 1.5 to 2 iterations. The latter can be seen from Fig. 2(b), where the average number of iterations for High SNR ET and Optimal ET are shown. The value for Optimal ET is again a lower bound, as it is the lowest possible average number of iterations, that can be achieved by a Low SNR ET scheme without having negative impact on the BLER.

It can be noticed, that in case of the TU channel the convergence of the decoding process is relatively fast, compared to a classical Turbo coding scenario over an AWGN channel, where convergence takes more iterations. The same behavior has been observed for other typical fading channels like PedestrianA, VehicularA, etc. The conclusion from this observation is that

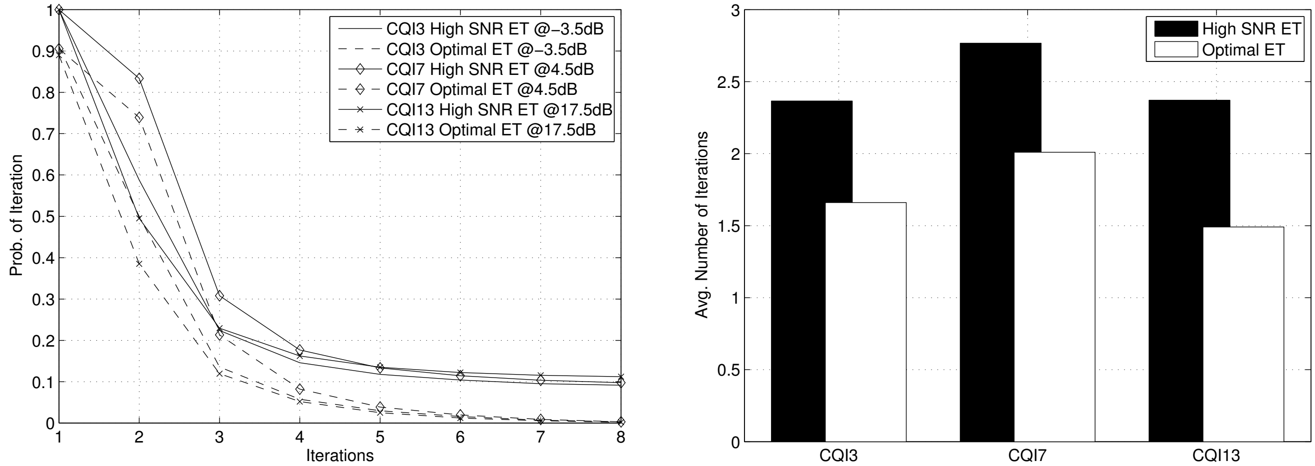
- for decodable blocks, i.e. in 90% of the cases, the decoder executes only about 1.5 to 2 iterations, but
- for undecodable blocks, i.e. in 10% of the cases, the decoder executes the full number of $i_{\max} = 8$ iterations.

This means that although there are only 10% undecodable blocks, their influence on the overall decoding effort is about four times higher than the influence of the decodable blocks. It is therefore easy to see, that Low SNR ET offers high potential for reducing the decoding effort and with it the power consumption and latency of the decoder.

III. Low SNR Early Termination

A. Low SNR ET Scheme

The basic principle of the proposed Low SNR ET scheme is described as follows: During the iterative Turbo decoding



(a) Probability of iteration.

(b) Averaged iterations.

Fig. 2. Probability that an iteration is executed and average number of iterations at SNR corresponding to a BLER of 10%. Comparison of High SNR ET and Optimal ET for different CQIs.

process the weight w_i associated with a specific decoder output sequence is computed after each iteration i. The weight w_i can be computed based on different criteria and should provide a measure for the convergence of the decoding process. It is always selected such that it is expected to decrease with ongoing iterations, in case of decodeable blocks. Based on w_i Low SNR ET can be realized by stopping the iteration process

- if $w_1 > c_{th}$, i.e. if the weight is greater than a predefined threshold c_{th} after the first iteration, or
- if $w_i > w_{i-1} + \delta$ for $i = 2 \dots i_{\max} - 1$, i.e. if the weight after the i-th iteration is greater than the weight after the previous iteration plus some design parameter $\delta \geq 0$.

In this case the current code block is considered to be undecodable and retransmission has to be issued. It can be directly seen, that each wrong decision has a negative impact on the BLER and will lead to unnecessary retransmissions. Therefore it is important to select the computation of w_i and the values for c_{th} and δ in such a way, that the BLER impact at 10% is negligible, but the ET scheme is still efficient.

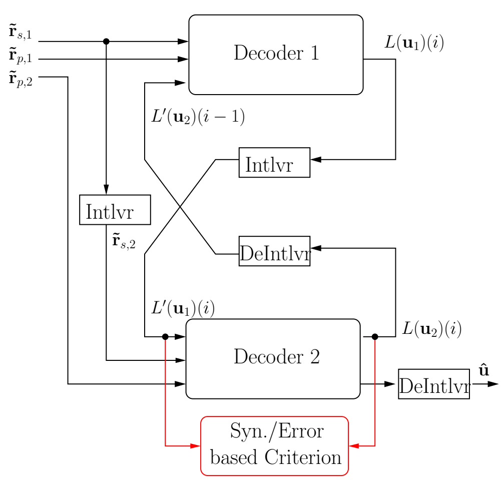


Fig. 3. Turbo decoding framework and extension for syndrome- or error-based criteria.

Different decoder output sequences can be considered for the computation of w_i. Here, the syndrome weight, the weight of the estimated error sequence and the inverse sum of the absolute likelihoods of systematic bits are considered. The first two criteria are solely based on hard decision computations, while the third one, which was similarly used in [10], is based on soft decision values.

Before these three possibilities are described in detail, the Turbo decoding framework is briefly reviewed. As shown in Fig. 3, two constituent decoders are assumed. In each iteration i, each decoder gets received softbits and a priori bits as inputs, and produces extrinsic LLRs of the systematic bits as output. The received softbits are denoted by $\tilde{\mathbf{r}}_1$ and $\tilde{\mathbf{r}}_2$ for the first and second constituent decoder, respectively. They consist of a systematic part $\tilde{\mathbf{r}}_{s,1}$ and $\tilde{\mathbf{r}}_{s,2}$ and a parity part $\tilde{\mathbf{r}}_{p,1}$ and $\tilde{\mathbf{r}}_{p,2}$. The a priori information $L'(\mathbf{u}_2)(i-1)$ of the first constituent decoder is the deinterleaved extrinsic LLR $L(\mathbf{u}_2)(i)$, which is generated by the second constituent decoder. The same holds for the apriori LLR $L'(\mathbf{u}_1)(i)$ of the second constituent decoder, which is the interleaved extrinsic LLR $L(\mathbf{u}_1)(i)$ generated by the first constituent decoder. Additionally the second constituent decoder produces after deinterleaving the a posteriori estimate $\hat{\mathbf{u}}$ of the original systematic bits $\mathbf{u}$.

It is assumed, that w_i is computed after a full iteration, i.e. after the second constituent decoder. The three criteria can then be computed as follows ($w_H(\cdot)$ denotes hamming weight in the following and L_x the length of sequence x):

- **Syndrome weight**: $w_i = \frac{1}{L_b} w_H(\mathbf{b}(i))$
  The syndrome sequence $\mathbf{b}(i)$ is computed based on the estimated systematic and parity bits:
  - The systematic part is the hard decision of $L(\mathbf{u}_2)(i)$.
  - The parity part is computed by encoding the hard decision of $L'(\mathbf{u}_1)(i)$ and taking only the parity part of the resulting code sequence.
  - The combination of both parts forms the estimated sequence $\hat{\mathbf{v}}(i)$.
  - The syndrome is then computed as $\mathbf{b}(i) = \hat{\mathbf{v}}(i)\mathbf{H}^T$.

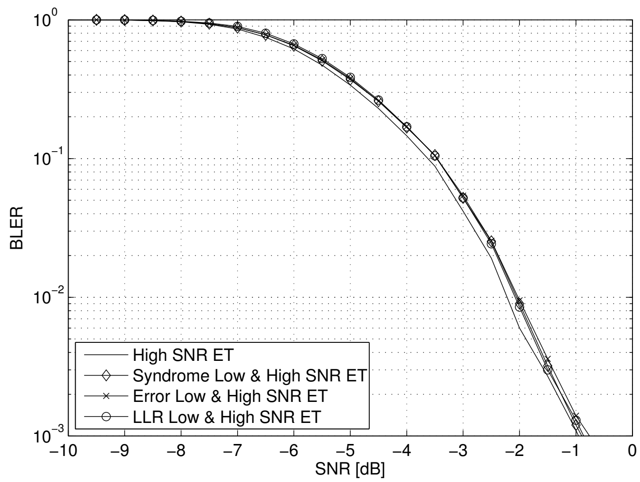


Fig. 4. Comparison of BLER for High SNR ET and combined Low and High SNR ET (CQI3).

Note that w_i typically decreases from 0.5 to 0 in case of convergence of the decoding process. The syndrome former $\mathbf{H}^T$ is orthogonal to the underlying code and can be realized using simple XOR-computations (see e.g. [13] for details on syndrome former computation).

- **Weight of estimated error**: $w_i = \frac{1}{L_e} w_H(\hat{\mathbf{e}}(i))$
  The error sequence $\hat{\mathbf{e}}(i)$ is found by modulo-2 addition of the encoded a priori and extrinsic sequences:
  - The encoded hard decision of $L'(\mathbf{u}_1)(i)$ is denoted as $\hat{\mathbf{v}}_1(i)$.
  - The encoded hard decision of $L(\mathbf{u}_2)(i)$ is denoted as $\hat{\mathbf{v}}_2(i)$.
  - The error sequence is $\hat{\mathbf{e}}(i) = \hat{\mathbf{v}}_1(i) \oplus \hat{\mathbf{v}}_2(i)$, where $\oplus$ denotes modulo-2 addition.
- **Inverse sum of absolute extrinsic LLRs**:
  $w_i = \frac{1}{\frac{1}{L_{u_2}} \sum |L(\mathbf{u}_2)(i)|}$

One of these criteria can be selected to realize the Low SNR ET scheme. The first two criteria depend on analyzing the inputs and outputs of the second constituent decoder, as shown in Fig. 3, while the last criterion is based only on the outputs of the decoder. However, the advantage of the first two criteria is, that, besides the division by the length, their computation involves only operations on bits (counters and XOR). All three can be realized without modifying the decoding algorithm itself, but can be generated from the available outputs.

It can be seen, that all criteria will tend to zero in case of a decodable block. The termination condition $w_i > w_{i-1} + \delta$ is based on this property.

The condition used after the first iteration, $w_1 > c_{th}$, is based on the observation that even after the first iteration, the weight w_i can be used to distinguish between decodable and undecodable blocks with certain probability. The required threshold c_{th} can be found by generating statistics about weights of decodable and undecodable blocks and selecting the value c_{th} as a suitable trade-off between possible wrong decisions and expected reduction of computational effort.

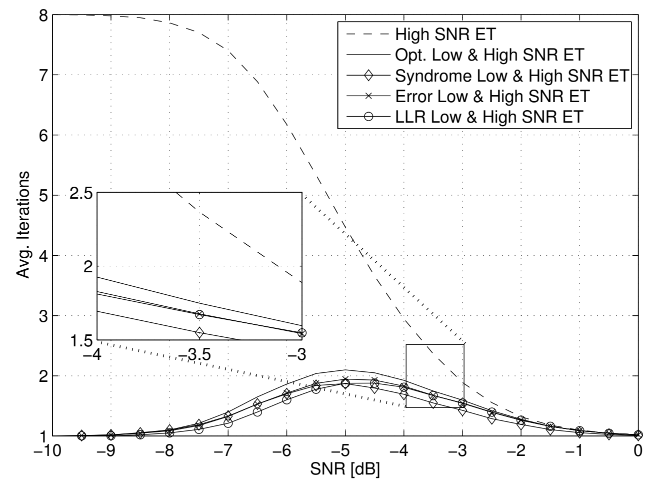


Fig. 5. Comparison of the average number of iterations for High SNR ET and for combined Low and High SNR ET (CQI3).

For the LLR based criterion, the threshold depends on various factors like current channel conditions and characteristics of the preceding stages, including equalization, demapping, quantization. This makes a general selection of c_{th} for the LLR based criterion difficult. On the other hand, for the hard-decision criteria the choice of c_{th} primarily depends on the CQI and is much less sensitive to other system parameters. Therefore for these criteria it is sufficient to pre-compute and store adequate c_{th} values for each MCS value, and select c_{th} solely based on the current MCS value during decoder operation.

B. Evaluation in Typical LTE Scenarios

The efficiency of the described Low SNR ET scheme is now discussed for the system parameters given in Sec. II. High SNR ET is always realized using the CRC24, as specified in LTE [14].

As described, the selection of the threshold c_{th} is the most important design factor. Its choice directly influences the complexity reduction, but also the probability of a false decision: A smaller value of c_{th} results in a higher probability of stopping the decoding process. On the one hand this increases the chances of identifying undecodable blocks and thus reduces the average number of iterations, but on the other hand it also increases the risk of stopping the decoding process for blocks which are actually decodable and thus increasing the BLER. Therefore, the threshold c_{th} has to be selected given an acceptable BLER loss. For the simulation results presented in this section, a maximum acceptable loss of $0.2dB$ is chosen.

The second design parameter δ is set to zero. It is easy to see that its influence is small, if the average number of iterations is also small, as described in Sec. II.

Fig. 4 compares the BLER for CQI3 of High SNR ET, which is the reference BLER, and the BLERs for the combination of High SNR ET and the proposed Low SNR ET approach using the three different criteria. It can be observed, that the BLER is slightly degraded due to the selection of c_{th} – however, the degradation is smaller than $0.2dB$ in all three cases.

TABLE I
SUMMARY OF RESULTS FOR LOW SNR ET FOR DIFFERENT CQIS AND DIFFERENT CRITERIA (SIMULATION PARAMETERS AS GIVEN IN SEC. II).

	Code Rate	Mod. Order	Block Length	δ	High SNR ET Avg. It.	**LLR based** Low & High SNR ET c_{th} / Avg. It. / Reduction	**Error based** Low & High SNR ET c_{th} / Avg. It. / Reduction	**Syndrome based** Low & High SNR ET c_{th} / Avg. It. / Reduction
CQI3	0.19	QPSK	3016	0	2.4	0.419 / 1.7 / 29%	0.161 / 1.7 / 29%	0.365 / 1.5 / 38%
CQI7	0.37	16QAM	5904	0	2.8	0.471 / 2.0 / 29%	0.178 / 2.1 / 25%	0.398 / 1.9 / 32%
CQI13	0.75	64QAM	4344	0	2.4	0.452 / 1.5 / 38%	0.234 / 1.7 / 29%	0.196 / 1.5 / 38%

The average number of iterations depending on the SNR is shown in Fig. 5. For a system with High SNR ET, the number of iterations is maximum ($i_{\max} = 8$ in this case) in the low SNR range, because in this range most of the blocks are undecodable. For increased SNR, the percentage of decodable blocks increases, such that the average number of iterations tends to one. If Low SNR ET is employed additionally, the average number of iterations in the low SNR range is decreased, because processing of undecodable blocks is mostly avoided. This can be seen in Fig. 5, which shows the result of the three Low SNR ET approaches and compares them to optimal Low SNR ET. It is easy to see, that all three schemes perform well and resemble the optimal Low SNR ET curve. Because of tolerating a small loss of BLER performance, the average number of iterations is actually even slightly lower.

A closer inspection at the working point of -3.5dB reveals that a decoder with CRC24 based High SNR ET, but without Low SNR ET, executes about 2.4 iterations in average for the given simulation parameters. If Low SNR ET is employed in addition, the average number of iterations is reduced to about 1.5 and 1.7, for the syndrome based criterion and for the LLR or error based criteria, respectively. Thus the syndrome based criterion reduces the average number of iterations by more than one third.

Table I summarizes parameter settings and numerical results of the three criteria for CQI3, CQI7, and CQI13. For each CQI, it lists the used threshold c_{th} along with the average number of iterations for High SNR ET, the combination of Low and High SNR ET, and the resulting reduction of iterations in percent for each criterion. It can be seen, that for all CQIs a reduction of more than 30% is achieved, if the syndrome based criterion is employed. The LLR based criterion yields a reduction of iterations, that is smaller or equal to the syndrome based criterion. Its computation is however based on soft decision values and is therefore more complex than the syndrome based criterion.

Only results for the TU channel are presented in this paper. However, the performance of the proposed approach has also been verified for other fading channel models and also for transmission over an AWGN channel.

IV. CONCLUSION

A Low SNR ET scheme has been described in this paper and its efficiency has been verified using a typical LTE transmission scenario. It has been explained that Low SNR ET is especially effective if the average number of iterations of the Turbo decoder is relatively small. As this is the case for typical mobile transmission channels, represented by e.g. the TU channel model, Low SNR ET becomes an important factor for the reduction of computational complexity of Turbo decoding in LTE.

For the realization of Low SNR ET, different criteria have been analyzed. Selecting the criterion as the syndrome weight of a combination of decoder output and a priori input, has been shown to be very effective: While it outperforms an LLR based criterion in terms of reduction of the average iteration number, its computation is significantly easier, because only hard decision values are involved. For the different CQI values a reduction of the average number of iterations of about one third has been observed. This does not only offer a significant potential for reduction of power consumption, but also decreases the latency of the Turbo decoder.

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