How to Improve OFDM-like Data Estimation by Using Weighted Overlapping

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***Abstract*— OFDM based transmission systems efficiently compute least square estimates of the transmitted data in a blockwise manner. However, there is a need to frequently insert guard periods in the transmitted data, which may significantly decrease the throughput of the transmission. We present weighted overlapping methods that compute data estimates efficiently in a blockwise manner without the need for inserted guard periods. Applying the methods in a wireless transmission scenario leads to a bit error rate performance that is over a wide range of E_{B}/N_0 values similar to the one obtained if exact least square (LS), or alternatively, minimum mean square error (MMSE) data estimation is applied. However, the weighted overlapping methods may require significantly less computational resources than the exact LS/MMSE estimation of data transmitted without guard periods.**

***Index Terms*— OFDM, block processing, guard period, LS, MMSE,linear data estimation**

I. Introduction

ORTHOGONAL Frequency Domain Multiplexing (OFDM) based transmission systems are well-known for their computational efficiency in equalizing the wireless broadband channel. For this reason, in particular, they are used in many high speed wireless communication systems [3], [5], [6]. If we describe OFDM based block transmissions using a linear data model, different concepts are found that contribute to OFDM's low complexity: Data blocking is used to create small circulant structured channel submatrices, least square estimates of the transmitted data are computed blockwise on the basis of the small submatrices, and the eigenvalue decomposition (EVD) of the circulant submatrices is efficiently applied [7], [8], [9]. Based on these concepts, we may establish further block transmission systems that show the same complexity as OFDM, but may perform differently or even better in many technical respects [7]. However, these block transmission systems (including OFDM) require the frequent insertion of guard periods into the transmitted data. In the WLAN standards [3], [5], 20% of the total transmission time is allocated to the guard periods. In the DRM standard [4], a mode is established that even allocates 44%. On the one hand, the guard periods enable the blockwise computation of LS/MMSE data estimates. On the other hand, they significantly reduce the throughput of the transmission and increase the required transmission energy to obtain similar E_{B}/N_0 ratios at the receiver. If the guard periods are removed we usually have to process all the received data (of one data burst) instead of only a small block (one OFDM symbol) to compute a LS/MMSE estimate of one transmitted data symbol. This approach would increase the demands on the computational resources enormously, but it would also avoid the need for guard periods.

Here, we present methods, named weighted overlapping, that enable the blockwise computation of data estimates even if no guard periods are inserted into the transmitted data. Their required computational resources (number of multiplications, storage space, processing structure) are much smaller than the ones which are necessary for the exact LS/MMSE estimation, when no guard periods are inserted. Although they will require higher computational resources than the data estimation in OFDM systems, the total requirement lies in the same dimension (and guard periods are not required). Weighted overlapping exploits an error distribution that emerges when the guard periods are removed. Over a wide range of E_{B}/N_0 values it shows a similar bit error rate performance as the usage of exact LS/MMSE estimation.

Section II presents a data model that is used to discuss the underlying concepts that lead to OFDMs low complexity. Furthermore, the removal of guard periods and its effect on the LS/MMSE estimation is analyzed and described by statistical means. In Section III, it is shown how the statistical properties may be exploited by using weighted overlapping.

Then, in Section IV, we include weighted overlapping in a wireless transmission scenario and compare its bit error rate performance with existing estimation methods. Conclusions are drawn in Section V.

II. Data Model

A data vector $\boldsymbol{d} \in \mathbb{C}^{JB}$ is to be transmitted over a time dispersive wireless channel that is described by a channel vector $\boldsymbol{h} \in \mathbb{C}^{L}$. On the channel, which is assumed to be time invariant during the transmission of the data vector $\boldsymbol{d}$, a noise vector $\boldsymbol{n} \in \mathbb{C}^{JB+L-1}$, which is obtained by sampling a white Gaussian noise process with power σ^2, is additively superimposed. The received vector $\boldsymbol{x} \in \mathbb{C}^{JB+L-1}$ can then be computed according to

$$\boldsymbol{x} = \boldsymbol{H}\boldsymbol{d} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{H} \in \mathbb{C}^{(JB+L-1)\times(JB)}$ denotes the channel convolution matrix. The data vector $\boldsymbol{d}$ consists of J data blocks $\boldsymbol{d}^{(j)} \in \mathbb{C}^{B}$ that are arranged amongst one another. The receiver shall compute linear estimates $\hat{\boldsymbol{d}}$ of the transmitted data vector $\boldsymbol{d}$ by using perfect channel state information (knowledge of vector $\boldsymbol{h}$), an estimate of the power σ^2 and the received vector $\boldsymbol{x}$. Assuming uncorrelated data $\boldsymbol{d}$ and noise $\boldsymbol{n}$ and an autocorrelation of the data according to $E\{\boldsymbol{d}\boldsymbol{d}^H\} = \boldsymbol{I}$, where $\boldsymbol{I}$ denotes an identity matrix, MMSE estimates of the transmitted data are obtained by

$$\hat{\boldsymbol{d}}_{\text{mmse}} = \left(\boldsymbol{H}^H\boldsymbol{H} + \sigma^2\right)^{-1}\boldsymbol{H}^H\boldsymbol{x}. \tag{2}$$

Setting the noise power σ^2 to zero in this equation yields LS estimates of the transmitted data. The LS/MMSE estimators

$$\boldsymbol{E}_{\text{mmse}} = \left(\boldsymbol{H}^H\boldsymbol{H} + \sigma^2\right)^{-1}\boldsymbol{H}^H, \tag{3}$$

$$\boldsymbol{E}_{\text{ls}} = \boldsymbol{E}_{\text{mmse}}|_{\sigma=0} \tag{4}$$

may also be considered as channel equalizers, as they remove the inter symbol interference caused by the time dispersive channel exactly ($\boldsymbol{E}_{\text{ls}}\boldsymbol{H} = \boldsymbol{I}$) or approximately ($\boldsymbol{E}_{\text{mmse}}\boldsymbol{H} \approx \boldsymbol{I}$).

In high speed wireless communications, only a very short time period is available to compute the estimator and the data estimates. This makes the above approach inappropriate, particularly if the data vector exceeds a certain length. If this occurs, the matrices involved in the computation would become huge as would the computational complexity and the required storage space. This problem may be solved by using the concept of data blocking, i.e. inserting guard periods between the data blocks $\boldsymbol{d}^{(j)}$ of the transmitted data vector $\boldsymbol{d}$. If the guard periods are long enough, each data block can be estimated separately on the basis of a significantly smaller estimator matrix. In addition, the same estimator matrix may be used for the estimation of all data blocks that belong to the same channel vector.

There are different kinds of guard periods, that are proposed for OFDM and related systems: The cyclic prefix, the zero pads, and the known symbol pads [1], [2], [7], [8]. Although they perform differently in various technical respects, essentially, they all convert the huge channel convolution matrix $\boldsymbol{H}$ of equation (1) into either circulant or rectangular structured independent submatrices [7].

Zero padding inserts $L-1$ between subsequent data blocks. This leads to independent rectangular structured channel submatrices $\bar{\boldsymbol{H}}_B$ of size $B+L-1 \times B$, which we illustrate in the following example ($J = B = L = 2$):

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} h_1 & & & & \\ h_2 & h_1 & & & \\ & h_2 & h_1 & & \\ & & h_2 & h_1 & \\ & & & h_2 & h_1 \\ & & & & h_2 \end{bmatrix} \begin{bmatrix} d_1^{(1)} \\ d_2^{(1)} \\ 0 \\ d_1^{(2)} \\ d_2^{(2)} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{bmatrix}$$

$$= \left[\begin{array}{cc|cc} h_1 & & & \\ h_2 & h_1 & & \\ & h_2 & & \\ \hline & & h_1 & \\ & & h_2 & h_1 \\ & & & h_2 \end{array}\right] \begin{bmatrix} d_1^{(1)} \\ d_2^{(1)} \\ d_1^{(2)} \\ d_2^{(2)} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{bmatrix}.$$

If we add the first and the third and also the fourth and sixth received symbols, we may obtain independent circulant structured channel submatrices $\tilde{\boldsymbol{H}}_B$ of size $B \times B$:

$$\begin{bmatrix} x_1 + x_3 \\ x_2 \\ x_4 + x_6 \\ x_5 \end{bmatrix} = \left[\begin{array}{cc|cc} h_1 & h_2 & & \\ h_2 & h_1 & & \\ \hline & & h_1 & h_2 \\ & & h_2 & h_1 \end{array}\right] \begin{bmatrix} d_1^{(1)} \\ d_2^{(1)} \\ d_1^{(2)} \\ d_2^{(2)} \end{bmatrix} + \begin{bmatrix} n_1 + n_3 \\ n_2 \\ n_4 + n_6 \\ n_5 \end{bmatrix}.$$

After data blocking, the estimation may be performed on the basis of the small subsystems of equations:

$$\boldsymbol{x}_{\text{R}}^{(j)} = \bar{\boldsymbol{H}}_B\boldsymbol{d}_{\text{R}}^{(j)} + \boldsymbol{n}_{\text{R}}^{(j)}, \boldsymbol{x}_{\text{C}}^{(j)} = \tilde{\boldsymbol{H}}_B\boldsymbol{d}_{\text{C}}^{(j)} + \boldsymbol{n}_{\text{C}}^{(j)}. \tag{5}$$

We use the subscript R and C to denote the rectangular and circulant case respectively. In the case of zero padded OFDM, an IDFT matrix $\boldsymbol{F}_B^H$ is additionally multiplied from the left to each transmitted data block (subscript F):

$$\boldsymbol{x}_{\text{F}}^{(j)} = \tilde{\boldsymbol{H}}_B\boldsymbol{F}_B^H\boldsymbol{d}_{\text{C}}^{(j)} + \boldsymbol{n}_{\text{C}}^{(j)}. \tag{6}$$

In the circulant case, we may apply the eigenvalue decomposition (EVD) to further decrease the computational complexity of the data estimation. The EVD is given by

$$\tilde{\boldsymbol{H}}_B = \boldsymbol{F}_B^H\boldsymbol{D}_B\boldsymbol{F}_B, \tag{7}$$

where $\boldsymbol{F}_B$ and $\boldsymbol{F}_B^H$ are the DFT and IDFT matrices of size $B \times B$. The diagonal matrix $\boldsymbol{D}_B$ contains the eigenvalues of $\tilde{\boldsymbol{H}}_B$. They may be easily computed by performing one DFT from the first column $\tilde{\boldsymbol{H}}_B(:,1)$ of $\tilde{\boldsymbol{H}}_B$. When we arrange the eigenvalues on a diagonal using the function 'diag', the diagonal matrix $\boldsymbol{D}_B$ may be described as

$$\boldsymbol{D}_B = \text{diag}\left(\boldsymbol{F}_B \tilde{\boldsymbol{H}}_B(:,1)\right). \tag{8}$$

By applying the LS/MMSE estimation and the EVD of circulant matrices to compute the estimators on the basis of the subsystems (5) and (6), we obtain the following estimators:

$$\boldsymbol{E}_{\text{C,ls}} = \tilde{\boldsymbol{H}}_B^{-1} = \boldsymbol{F}_B^H \boldsymbol{D}_B^{-1} \boldsymbol{F}_B, \tag{9}$$

$$\boldsymbol{E}_{\text{C,mmse}} = \boldsymbol{F}_B^H \underbrace{\left(\boldsymbol{D}_B^H \boldsymbol{D}_B + \sigma_{\text{C}}^2 \boldsymbol{I}\right)^{-1} \boldsymbol{D}_B^H}_{\boldsymbol{D}_{B,\text{mmse}}^{-1}} \boldsymbol{F}_B, \tag{10}$$

$$\boldsymbol{E}_{\text{F,ls}} = \boldsymbol{D}_B^{-1} \boldsymbol{F}_B. \tag{11}$$

The existence of the LS estimator in the rectangular case is guaranteed, whereas in the circulant case, the estimator might not exist. However, the MMSE estimator is guaranteed to exist in both cases. The transmission systems that use the above estimators $\boldsymbol{E}_{\text{F,ls}}$ and $\boldsymbol{E}_{\text{F,ls}}$ are depicted in Figure 1. They are known as ZP-OFDM and C-SC ('Circulant' Single Carrier). The DFT may be implemented using FFT algorithms. Both systems show the same low complexity in computing the data estimates. The price paid for this computational efficiency is the need to frequently insert guard periods in the transmitted data. This may effect the required transmission energy and the data throughput detrimentally (not all the time is allocated to transmit information data).

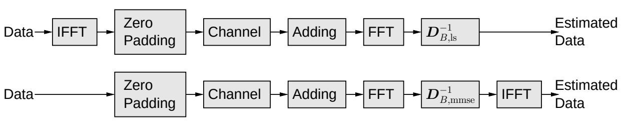

Fig. 1. Data estimation in ZP-OFDM and C-SC transmission systems applying zero padding.

What happens to the estimation quality if we retain the efficient estimation but remove the guard periods? To answer this question we first consider the approximation error that results from this procedure. The large channel convolution matrix $\boldsymbol{H}$ of equation (1) may be decomposed into a block diagonal matrix $\boldsymbol{I}_J \otimes \tilde{\boldsymbol{H}}_B$ that consists of independent circulant submatrices $\tilde{\boldsymbol{H}}_B$ and an error matrix $\boldsymbol{H}_{\text{E}}$ ('$\otimes$' denotes the Kronecker product):

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{I}_J \otimes \tilde{\boldsymbol{H}}_B \\ \boldsymbol{0}_{(L-1)\times JB} \end{bmatrix} + \boldsymbol{H}_{\text{E}}. \tag{12}$$

Applying this decomposition to equation (1) and performing blockwise data estimation using the estimator according to equation (9) (assuming the estimator exists) yields:

$$\begin{aligned} \hat{\boldsymbol{d}} &= \begin{bmatrix} \boldsymbol{I} \otimes \boldsymbol{E}_{\text{C,ls}} & \boldsymbol{0}_{JB\times(L-1)} \end{bmatrix} \boldsymbol{x} & (13) \\ &= \boldsymbol{d} + \begin{bmatrix} \boldsymbol{I} \otimes \tilde{\boldsymbol{H}}_B^{-1} & \boldsymbol{0}_{JB\times(L-1)} \end{bmatrix} \boldsymbol{H}_{\text{E}} \boldsymbol{d} & (14) \\ &\quad + \begin{bmatrix} \boldsymbol{I} \otimes \tilde{\boldsymbol{H}}_B^{-1} & \boldsymbol{0}_{JB\times(L-1)} \end{bmatrix} \boldsymbol{n}. \end{aligned}$$

In this case the data estimates can be decomposed into the true data, an estimation error (resulting from not inserting guard periods) and a noise term. The absolute estimation error is therefore given by

$$\boldsymbol{e}_{\text{C}} = \text{abs}\left(\begin{bmatrix} \boldsymbol{I} \otimes \tilde{\boldsymbol{H}}_B^{-1} & \boldsymbol{0}_{JB\times(L-1)} \end{bmatrix} \boldsymbol{H}_{\text{E}} \boldsymbol{d}\right). \tag{15}$$

We may also compute the absolute estimation error that results from removing guard periods in the case of rectangular submatrices and also for MMSE estimation (instead of LS), see [7].

For a channel vector of dimension $L = 17$ that describes a multi path channel, BPSK modulated data, and a block size $B = 64$, we compute an absolute estimation error as a function of the time index for the cases 'CIR-64' (LS estimation on basis of circulant submatrices) and 'REC-64' (LS estimation on basis of rectangular submatrices). By averaging over multiple channel realizations, we obtain the ensemble-averaged absolute estimation errors as a function of the time index according to case 'CIR-64' and case 'REC-64' Figure 5. They show similar behavior. The averaged approximation errors are large at the beginning and end of a data block and small in the middle. The histograms of occurring absolute estimation errors at the beginning, at a quarter and at the middle of one data block are depicted in Figure 2. They show a lower variance of the estimation error in the middle of a block compared to the beginning and the quarter.

III. Weighted Overlapping

How can we exploit these characteristics of the approximation error? One way is to perform the data estimation on the basis of overlapping subsystems, see Figure 3. We compute the estimated data by multiplying each overlapping received block ($\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}$, and $\boldsymbol{x}^{(3)}$) from the left by the small estimators. In this way, we obtain the overlapping estimated data blocks $\hat{\boldsymbol{d}}^{(1)}, \hat{\boldsymbol{d}}^{(2)}$, and $\hat{\boldsymbol{d}}^{(3)}$. The estimation error of each block follows the distribution discussed in Section II. Depending on the selected block size and the size of the overlap, we may obtain several

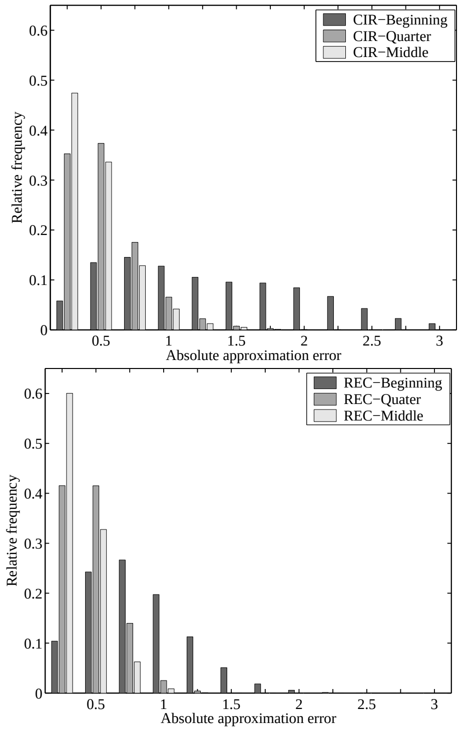

Fig. 2. Relative frequency of occurring absolute approximation errors for circulant and rectangular subsystems at different positions of a data block.

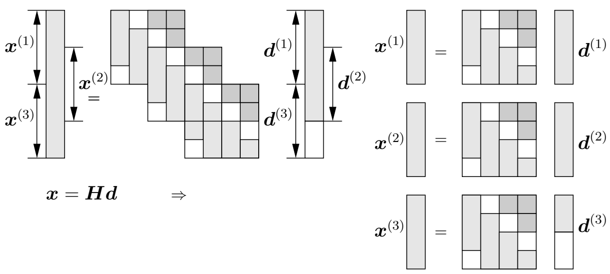

Fig. 3. The decomposition of a large channel convolution matrix into many circulant overlapping submatrices causes the approximation error as discussed in Section II.

estimates of each transmitted data symbol. Each estimate is in its block at a different position and thus follows a different error distribution. We may exploit the distribution, by discarding the values at the edges of each block (weighting by zero) and averaging the remaining estimates. The weights of each estimated symbol are depicted in Figure 4 for block size $B = 64$ and an overlapping increment of $S = 32$, $S = 16$, and $S = 8$, respectively. One box represents an 8×1 data vector. This is a heuristical way of finding good weights that take into account the large estimation errors at the edges and that multiplications by a power of $1/2$ can be implemented very efficiently by using shift operations. In addition, an analytical derivation of optimal weights would indeed be desirable (not in the scope of this paper). The ensemble-averaged absolute estimation error for the methods depicted in Figure 4 and also

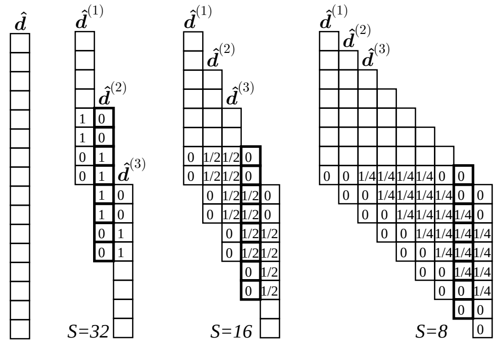

Fig. 4. Weights of the estimated data symbols for overlapping increments of $S = 32$, $S = 16$, and $S = 8$.

for $S = 4$ are depicted in Figures 5 for circulant and rectangular submatrices respectively. The first method ($S = 32$) cuts the small errors out of each block and pastes them together. In this way, each data symbol is estimated at the lower error level (avoiding the edges). Additional weighting and adding of the estimated symbols (cases $S = 16$, $S = 8$, $S = 4$) may even further decrease the error.

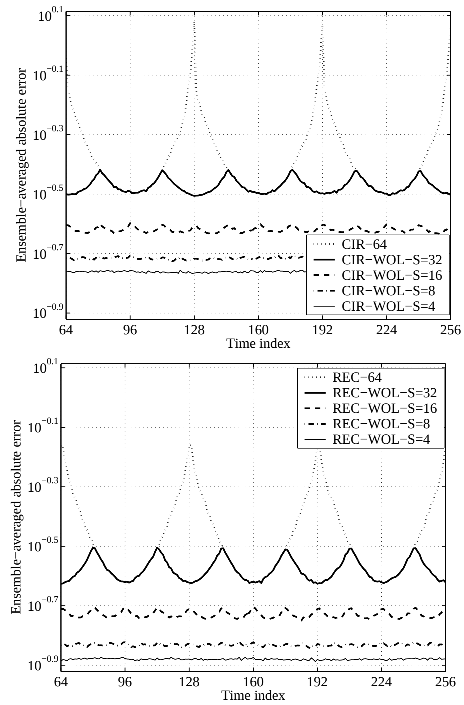

Fig. 5. Ensemble-averaged estimation error as a function of the time index for different approximation methods relying on weighted overlapping (WOL) on the basis of circulant (CIR) or rectangular (REC) submatrices and different overlapping increments S.

IV. Simulations

So far we have analyzed the effects of weighted overlapping on the estimation error, however, we also need to consider the effect on the noise, see equation (14). Unfortunately, the averaging of the estimates does not reduce the noise influence markedly. This is a result of the Toeplitz (circulant case) or near Toeplitz (rectangular case) structure of the estimators $\boldsymbol{E}_{\text{C,ls,mmse}}$ and $\boldsymbol{E}_{\text{R,ls,mmse}}$, respectively. If we compute two estimates of one data symbol on the basis of two overlapping circulant submatizes of size $B \times B$ and set the overlapping increment $S = 1$, then $B - 1 = 63$ of all 64 multiplications between one row of the estimator and the respective received block are identical. This may be exploited by an updating algorithm that computes a second estimated overlapping data block on the basis of the first and a few further multiplications, however, it implies that the noise part of each estimate of one data symbol can be expected to be very similar and therefore averaging over the estimate symbols does not significantly reduce the influence of the noise. However, as shown in Section III, it does reduce the influence of the estimation error.

We simulate the bit error rates for BPSK modulated data, a $L = 17$ tap channel, whose taps are Rayleigh distributed and spaced by the symbol duration. We use MMSE estimation on the basis of circulant submatrices. One reference system uses a block size $B = 64$ without overlapping (case 'CIR-64'). It therefore measures the effect of the approximation error on the bit error rate. Another reference is the exact MMSE estimation on the basis of the large system of equations (1), after a circulant system has been established by adding the last $L-1$ rows to the first (case 'CIR-exactMMSE'). The other systems use weighted overlapping as estimation method. The weighting vectors discard the first and last 16 values of each data block and weight the remaining estimates of each data symbol equally (cases 'CIR-WOL-S=32', 'CIR-WOL-S=16', 'CIR-WOL-S=8', and 'CIR-WOL-S=4'). The respective BERs are depicted in Figure 6. Weighted overlapping may reach

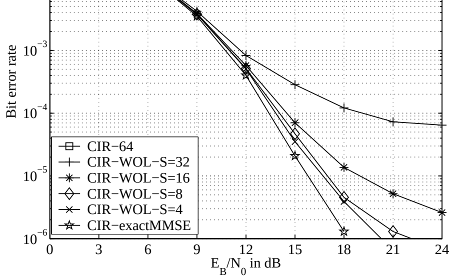

Fig. 6. BER performance of weighted overlapping in case of circulant submatrices compared to exact MMSE estimation.

over a wide range of E_{B}/N_0 ratios similar bit error rates than exact MMSE estimation. The advantage of weighted overlapping lies in the significantly reduced demands on computational resources, since instead of computing the huge estimator and use it to compute the estimates, only a small estimator is computed and reused in a blockwise manner. This concept is also used in OFDM and related systems. There, however, the blockwise estimation is enabled by inserting significant amount of guard periods that reduces throughput and enlarge the required transmission power.

V. Conclusions

Combining OFDM-like data estimation with weighted overlapping is an efficient method of computing data estimates in a blockwise manner. In contrast to OFDM-like estimation methods, it does not need the insertion of guard periods in the transmitted data to enable the blockwise data estimation. It can therefore improve the data throughput enormously while also demanding less transmission energy to obtain similar E_{B}/N_0 ratios at the receiver. Over a wide range of E_{B}/N_0 ratios, weighted overlapping may show similar BER performance as the exact estimator based on the huge channel matrix, but can be much more efficient in the use of computational resources.

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