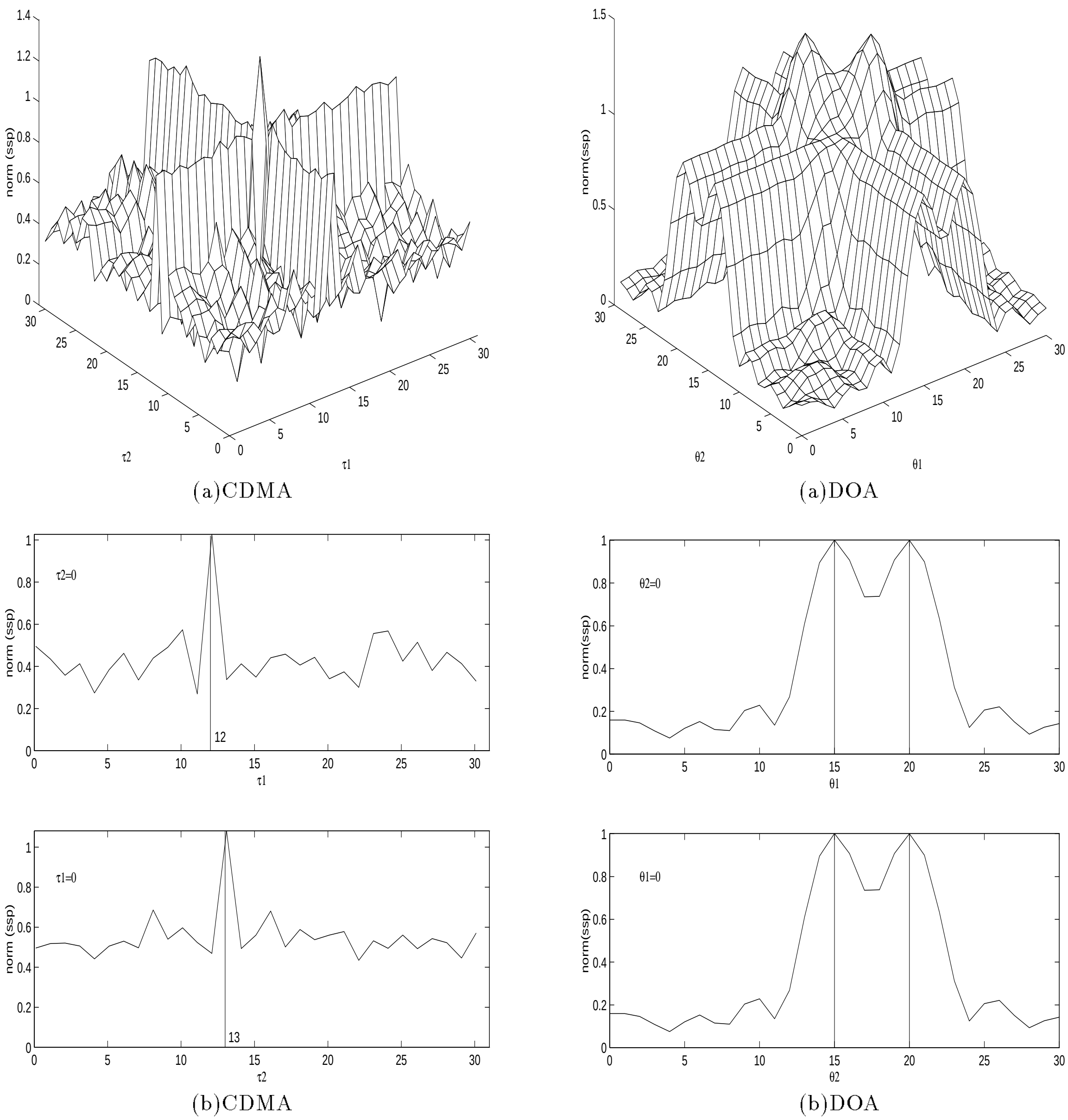


**Figure 1. (a) Two–dimensional optimization for CDMA delay estimation with $\tau_1 = 12$ and $\tau_2 = 13$ (left) and DOA–estimation with $\theta_1 = 15°$ and $\theta_2 = 20°$ (right). (b) Plots obtained from plot (a) for $\tau_2 = 0/\theta_2 = 0$ and $\tau_1 = 0/\theta_1 = 0$, respectively.**

## 5 CONCLUSION

In this paper the equivalence of signal and noise subspace methods for delay estimation in wireless CDMA systems has been shown. Based on this result a signal subspace method was presented. A signal subspace method is advantageous for a couple of reasons: computational complexity proportional to number of users, in many cases the signal subspace is smaller than the noise subspace, efficient (spherical) subspace methods can be derived for signal subspace methods. Finally, the subspace methods for CDMA delay estimation were compared to the subspace methods for DOA–estimation. The basic differences were presented and the advantage of the CDMA system model with respect to subspace methods was discussed.

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Being able to work with the signal subspace is important for different reasons. For the signal subspace method the computational complexity is proportional to the number of users $k$ while if using the noise subspace the complexity is proportional to $N-k$.In many cases $k \ll N$ holds and therefore a signal subspace algorithm is computationally more efficient than working with the noise subspace. In addition, as we will see from the next section, updating signal subspaces can be performed in elegant ways when the parameters being estimated are time varying.

### 4.2 Spherical Subspace Algorithms

An important point which makes working in the signal subspace advantageous is that it is possible to derive efficient subspace algorithms by averaging the noise subspace. In theory (with infinitely many data samples) the noise subspace is spherical, i.e. $\sigma_{k+1} = \sigma_{k+2} = \ldots = \sigma_N$. In practice (with finite number of data samples) this spherical shape of the noise subspace is reasonably well approximated. This fact can be used by averaging (sphericalizing) the noise subspace, such that the noise subspace is fully determined by a one–dimensional subspace. Therefore, the problem is reduced from a $N$–dimensional subspace computation problem to a $(k+1)$–dimensional subspace computation problem. These spherical subspace algorithms [16, 17, 18] are recursive such that the spherical property and the tracking possibilities can be efficiently combined.

### 4.3 Numerical Issues and Parallel Implementation: Subspace Angles and Projections

The above results and algorithms have been designed on the data level, i.e. without building correlation matrices or projection matrices at any point throughout the algorithm. Instead the algorithms work directly on the data matrices and by optimizing the subspace angles.

It is well known [14] that the computation of the SVD of the data matrix $\boldsymbol{Y} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H$ is numerically better conditioned than the computation of the eigenvalue decomposition of the correlation matrix $\boldsymbol{R} = \boldsymbol{Y}^H\boldsymbol{Y} = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}$ $(\boldsymbol{\Lambda} = \boldsymbol{\Sigma}^2)$, since $\mathrm{cond}(\boldsymbol{R}) = [\mathrm{cond}(\boldsymbol{Y})]^2$ where $\mathrm{cond}(\boldsymbol{X}) = \frac{\sigma_1(\boldsymbol{X})}{\sigma_N(\boldsymbol{X})}$ defines the condition number of a $m \times N$ matrix $\boldsymbol{X}$. The same numerical advantages apply to using the subspace angles instead of the projection matrices for the estimation task.

Furthermore, the algorithms formulated on the data level are much more amenable to a parallel implementation than the correlation matrix/projection matrix based algorithms. The matrix multiplications required to build these matrices contradict a straightforward pipelined parallel implementation [19, 20]. This fact becomes even more important if nonstationarities of the delay estimates need to be tracked. Therefore, all parallel implementations of subspace tracking algorithms are based on SVD–updating [21, 22, 23]. This also holds for the spherical signal subspace algorithms, i.e. they are based on spherical SVD–updating methods [17] if a parallel implementation is amenable.

### 4.4 Comparison to DOA–Estimation

CDMA delay estimation determines the locally optimum delays in each of the $N$ chips by a closed form expression. Then a search over the set of the $N$ determined local solutions is performed in order to find the optimum of these closed form solutions (i.e. the search is reduced to $N$ discrete values in the parameter space, while DOA–estimation requires a search over the entire parameter space). It is always possible to use the subspace which provides the lower computational complexity due to corollary 3.1 or to use the signal subspace in conjunction with subspace averaging, i.e. a spherical signal subspace method [15].

Another important point that makes the CDMA delay estimation very amenable to subspace methods is the linear independence of the users in the parameter space. This is best illustrated by using a two–dimensional DOA–estimation problem ($\theta_1, \theta_2$ unknown parameters) and a two–dimensional CDMA delay estimation problem ($\tau_1, \tau_2$ unknown parameters) as an example.

**Example 4.2 [Two–dimensional estimation problems]** In figure 1 the results for a two–dimensional CDMA delay estimation problem (left side) and a two–dimensional DOA–estimation problem (right side) are shown. The 2–norm of the diagonal elements of the diagonal matrices obtained in (10), i.e. $\mathrm{norm(ssp)} = \|\mathrm{diag}\left(\boldsymbol{\Sigma}(\boldsymbol{V}_S^H\boldsymbol{Q}_1)\right)\|_2$ is used.

CDMA: We use the same scenario as in example 4.1 but now with two users with delays $\tau_1 = 12$ and $\tau_2 = 13$. Here, we consider the multidimensional optimization problem, i.e. we consider the whole parameter space and orthonormalize the respective code matrix $\boldsymbol{A}$ before we determine the subspace angles. Figure 1 shows the results for the whole parameter space $(\tau_1, \tau_2)$ in the three dimensional plot ((a)CDMA) as well as the results obtained by reducing the problem to two one–dimensional problems in the two two–dimensional plots with $\tau_2 = 0$ and $\tau_1 = 0$, respectively ((b)CDMA). Obviously, it is possible to handle the two one–dimensional problems independently.

DOA: $K = 2$ signals are impinging on a $N = 31$ element sensor array. The SNR is 8dB and $L = 50$ samples are taken. Figure 1 shows the results for the whole parameter space $(\theta_1, \theta_2)$ in the three dimensional plot ((a)DOA) as well as the results obtained by reducing the problem to a one–dimensional problem in the two two–dimensional plots with $\theta_2 = 0$ and $\theta_1 = 0$, respectively ((b)DOA). Contrary to the CDMA case both of the two two–dimensional plots yield maxima at both unknown parameters, such that it is not possible to handle the estimation problems for the two unknown parameters independently. □

The K–dimensional CDMA delay estimation problem can be decomposed into $K$ one–dimensional problems. Since the codes guarantee linear independence searching the parameter space for $\tau_i$ is independent of the parameter space of $\tau_k$ $(k \neq i)$ and therefore we obtain only one maxima in the respective parameter space $\tau_i$. This is not the case for the DOA–estimation, since searching the parameter space $\theta$ hits all unknown parameters, i.e. for $K = 2$ we obtain two maxima in the $\theta_1$ direction independent of $\theta_2$ and two maxima in the $\theta_2$ direction independent of $\theta_1$. As the $\theta_k$ move spatially closer together, eventually the two maxima cannot be distinguished anymore. Note, that in the above examples the two delays were closely spaced while the two DOAs are spatially well seperated.

The code matrix (5) in the CDMA delay estimation case is based on a matrix $\boldsymbol{A}_0 = [\boldsymbol{a}_1, \ldots, \boldsymbol{a}_K]$ (the index 0 indicates that no delays are applied, yet, in contrast to (5)) containing codes such that the columns of $\boldsymbol{A}$ are linearly independent for all possible delays. Therefore, even if $\tau_i = \tau_k$ holds $\boldsymbol{A}$ is a well–conditioned matrix. The array steering matrix in the DOA–estimation case is a Vandermonde matrix (8) such that for $\theta_i = \theta_k$ the matrix $\boldsymbol{A}$ is rank–deficient. Therefore, no sensible estimates can be expected in the DOA–estimation case if $\theta_i$ and $\theta_k$ are the directions of two closely spaced signals $(\theta_i \approx \theta_k)$.

**Theorem 3.1 [CS–decomposition of $N \times p$ dimensional orthogonal matrix]** *Let $\boldsymbol{W} \in \mathcal{C}^{N\times p}$ have orthonormal columns and let $\boldsymbol{W}$ be partitioned as follows*

$$\boldsymbol{W} = \begin{array}{c} p \\ \left[\begin{array}{c} \boldsymbol{W}_1 \\ \boldsymbol{W}_2 \end{array}\right] \end{array} \begin{array}{c} \\ k \\ \ell \end{array} ,$$

*where $k + \ell = N$ and $k \geq p$. Then, there exist orthonormal matrices $\boldsymbol{Z}_{L1} \in \mathcal{C}^{k\times k}$, $\boldsymbol{Z}_{L2} \in \mathcal{C}^{\ell\times\ell}$, and $\boldsymbol{Z}_R \in \mathcal{C}^{p\times p}$ such that*

$$\left[\begin{array}{cc} \boldsymbol{Z}_{L1}^H & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z}_{L2}^H \end{array}\right]\left[\begin{array}{c} \boldsymbol{W}_1 \\ \boldsymbol{W}_2 \end{array}\right]\boldsymbol{Z}_R = \left[\begin{array}{c} \boldsymbol{Z}_{L1}^H\boldsymbol{W}_1\boldsymbol{Z}_R \\ \boldsymbol{Z}_{L1}^H\boldsymbol{W}_2\boldsymbol{Z}_R \end{array}\right] =$$

1. if $p \leq \ell$

$$= \begin{array}{c} p \\ \left[\begin{array}{c} \boldsymbol{C} \\ \boldsymbol{0} \\ \boldsymbol{S} \\ \boldsymbol{0} \end{array}\right] \end{array} \begin{array}{c} \\ p \\ k-p \\ p \\ \ell-p \end{array}$$

2. if $p > \ell$

$$= \begin{array}{c} \ell \quad\quad p-\ell \\ \left[\begin{array}{cc} \boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{S} & \boldsymbol{0} \end{array}\right] \end{array} \begin{array}{c} \\ p \\ k-p \\ p \\ \ell-p \end{array}$$

*Here, $\boldsymbol{C}$ and $\boldsymbol{S}$ are nonnegative diagonal matrices satisfying*

$$\boldsymbol{C}^2 + \boldsymbol{S}^2 = \boldsymbol{I}.$$

*Therefore, for the diagonal entries we obtain $c_i^2 + s_i^2 = 1$ such that we can make the interpretation $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$ for some angle $\theta_i$.*

**Proof:** A appealing proof of a more general form of theorem 3.1 is given in [13]. The present form of the theorem only treats the cases (matrix dimensions) that occur in our subsequent discussion of the subspace methods. □

**Corollary 3.1** *Let $\boldsymbol{A} = \boldsymbol{QR}$ be the QR– decomposition of the code matrix/array steering matrix $\boldsymbol{A}$ and let $\boldsymbol{Q} = [\boldsymbol{Q}_1\ \boldsymbol{Q}_2]$, where $\boldsymbol{Q}_1 \in \mathcal{C}^{N\times k}$ and $\boldsymbol{Q}_2 \in \mathcal{C}^{N\times\ell}$. $\boldsymbol{V} = [\boldsymbol{V}_S\ \boldsymbol{V}_N]$ be the decomposition of the row space $\boldsymbol{V} \in \mathcal{C}^{N\times N}$ as obtained by the SVD of the data matrix $\boldsymbol{Y} = \boldsymbol{U\Sigma V}^H$ into signal subspace $\boldsymbol{V}_S \in \mathcal{C}^{N\times k}$ and noise subspace $\boldsymbol{V}_N \in \mathcal{C}^{N\times\ell}$. Then,*

$$\boldsymbol{\Sigma}(\boldsymbol{V}_S^H\boldsymbol{Q}_1) = \left(\boldsymbol{I} - \boldsymbol{\Sigma}^2(\boldsymbol{V}_N^H\boldsymbol{Q}_1)\right)^{1/2}. \qquad (10)$$

*With $\sigma_i(\boldsymbol{V}_S^H\boldsymbol{Q}_1) = \cos\theta_i$ and $\sigma_i(\boldsymbol{V}_N^H\boldsymbol{Q}_1) = \sin\theta_i$ the angles $\theta_i$ define the angles between the respective signal/noise subspace and the parameter space.*

**Proof:** 1.) $\ell \geq k$: Using the orthonormal matrices $\boldsymbol{Q} = [\boldsymbol{Q}_1\ \boldsymbol{Q}_2]$ and $\boldsymbol{V} = [\boldsymbol{V}_S\ \boldsymbol{V}_N]$ we compute the orthonormal matrix $\boldsymbol{W}_{VQ}$:

$$\begin{aligned} \boldsymbol{W}_{VQ} = \boldsymbol{V}^H\boldsymbol{Q} &= [\boldsymbol{V}_S\boldsymbol{V}_N]^H[\boldsymbol{Q}_1\boldsymbol{Q}_2] = \\ &= \left[\begin{array}{cc} \boldsymbol{V}_S^H\boldsymbol{Q}_1 & \boldsymbol{V}_S^H\boldsymbol{Q}_2 \\ \boldsymbol{V}_N^H\boldsymbol{Q}_1 & \boldsymbol{V}_N^H\boldsymbol{Q}_2 \end{array}\right] \end{aligned}$$

Defining $\boldsymbol{W}_1 = \boldsymbol{V}_S^H\boldsymbol{Q}_1$ and $\boldsymbol{W}_2 = \boldsymbol{V}_N^H\boldsymbol{Q}_1$ we obtain

$$\boldsymbol{W} = \left[\begin{array}{c} \boldsymbol{W}_1 \\ \boldsymbol{W}_2 \end{array}\right] = \left[\begin{array}{c} \boldsymbol{V}_S^H\boldsymbol{Q}_1 \\ \boldsymbol{V}_N^H\boldsymbol{Q}_1 \end{array}\right].$$

Now, applying the CS–decomposition to $\boldsymbol{W}$ yields

$$\boldsymbol{C} = \boldsymbol{\Sigma}(\boldsymbol{V}_S^H\boldsymbol{Q}_1) \quad \text{and} \quad \boldsymbol{S} = \boldsymbol{\Sigma}(\boldsymbol{V}_N^H\boldsymbol{Q}_1).$$

and since $\boldsymbol{C}^H\boldsymbol{C} + \boldsymbol{S}^H\boldsymbol{S} = \boldsymbol{I}$ we obtain (10).

2.) $\ell < k$: By exchanging the oder of $\boldsymbol{V}_S$ and $\boldsymbol{V}_N$ in $\boldsymbol{V}$, i.e. using $\boldsymbol{V} = [\boldsymbol{V}_N\ \boldsymbol{V}_S]$ we can apply the same reasoning as in case 1) and then adjust the dimensions. □

## 4 DELAY ESTIMATION BY SUBSPACE ANGLE OPTIMIZATION

In this section we will elaborate on the result of corollary 3.1. A signal subspace algorithm for delay estimation is derived. In order to reduce the computational complexity a spherical version of the signal subspace method is discussed. The relation between the delay estimation in CDMA systems and the DOA–estimation in array processing is also elaborated.

### 4.1 Signal Subspace Algorithms

Corollary 3.1 shows that using the signal subspace or the noise subspace for the estimation process is equivalent. Therefore, in terms of computational complexity it is always advantageous to work with the smaller of these two subspaces. Of course, increasing the dimension of the noise subspace (i.e. increasing the spreading gain/number of sensors) while the dimension of the signal subspace (i.e. the number of users) is preserved, increases the accuracy of the estimated delays. The equivalence between using signal and noise subspace as stated in corollary 3.1 holds for fixed $N$.

Consequently, it is possible to derive a signal subspace algorithm which is equivalent to the noise subspace algorithm presented in [7]. It is necessary, however, to orthonormalize the desired users signal vectors (i.e. compute an orthonormal basis $\boldsymbol{Q}_1$ for the parameter space $\boldsymbol{A}$ given by the code matrix for a specific set of delay values) in order to fullfill the assumptions made in corollary 3.1. Since the delay estimation can be reduced to one–dimensional optimization problems, this only requires that the desired user's signal vector $\boldsymbol{u}_k(\tau)$, must be normalized (i.e. $\boldsymbol{u}_k(\tau) \leftarrow \boldsymbol{u}_k(\tau)/\|\boldsymbol{u}_k(\tau)\|_2$) before the subspace angle is determined. This modification is essential in order to be able to use the signal subspace. The closed form expression for estimating the timings within each chip based on the signal subspace can be derived in the same way as it is done in [7] for the method based on the noise subspace. Without the above mentioned orthonormalization of the parameter space the estimation algorithm is not applicable to the signal subspace. The following example illustrates this fact.

**Example 4.1 [Use of an orthonormal basis for parameter space]** A CDMA system with $K = 3$ users is considered, where the spreading gain is $N = 31$ (Gold code) and the interfering users are 20dB stronger than the desired user. The delay estimates are computed for a SNR of 8dB using a data window of length $L = 50$. Table 1 shows the results of the subspace methods (a) without (ortho)normalization of the parameter space and (b) with (ortho)normalization of the parameter space. Obviously, using a normalized parameter space (table 1b) the results using the signal and the noise subspace are identical. Without normalization (table 1a) only the noise subspace yields sensible results. □

| | *user k* | $k = 1$ | $k = 2$ | $k = 3$ |
|---|---|---|---|---|
| | $\tau_k$ *(excat)* | *16.8551* | *5.9488* | *5.6748* |
| *(a)* | $\tau_k$ *(ssp)* | *17.0000* | *6.0000* | *6.0000* |
| | $\tau_k$ *(nsp)* | *16.8406* | *5.9490* | *5.6741* |
| *(b)* | $\tau_k$ *(ssp)* | *16.8448* | *5.9490* | *5.6741* |
| | $\tau_k$ *(nsp)* | *16.8448* | *5.9490* | *5.6741* |

**Table 1. Estimated delays using the signal subspace (ssp) and the noise subspace (nsp). (a) No orthonormalization of the parameter space was applied. (b) The parameter space is orthonormalized before the subspace methods are applied.**

estimation problem.

## 2 SYSTEM MODEL

### 2.1 CDMA System

We assume a $K$-user CDMA network with all users using binary phase shift keying for both data and chip modulations. The received signal is a superposition of the $K$ signals in additive white Gaussian noise given by

$$r(t) = \sum_{k=1}^{K} \alpha_k s_k(t - \tau_k) + \eta_t \qquad -\infty < t < \infty\,. \quad (2)$$

Here $\alpha_k$ and $\tau_k$ are the attenuation and delay respectively corresponding to the $k^{th}$ user and $\eta_t$ is assumed to be white Gaussian noise with zero mean and a two-sided power spectral density of $\mathcal{N}_0/2$. The complex baseband representation for the $k^{th}$ user's transmitted signal, $s_k$, is given by

$$s_k(t) = \sqrt{2P_k}\, e^{j\phi_k} \sum_i b_k^{(i)} a_k(t - iT), \qquad (3)$$

where $P_k$ is the transmitted power, $\phi_k$ is the carrier phase relative to the local oscillator at the receiver, $b_k^{(i)} \in \{+1, -1\}$ is the transmitted symbol, $a_k(t)$ is the spreading waveform, and $T$ is the symbol duration. The spreading waveform, $a_k(t)$, can be given by $a_k(t) = \sum_{n=0}^{N-1} \Pi_{T_c}(t - nT_c)\, a_k^{(n)}$, where $\Pi_{T_c}(t)$ is a rectangular pulse, $T_c = T/N$, and $a_k^{(n)} \in \{+1, -1\}$.

The received continuous signal is discretized and vectorized by sampling the output of a chip matched filter and buffering $N$ samples to form one observation vector, $\boldsymbol{y}_i \in \mathcal{C}^N$. Since the system is asynchronous, each observation vector can be viewed as a linear combination of $2K$ signal vectors (2 components from each user) plus noise. The factors due to the power, phase, and transmitted symbols of the $k^{th}$ user are collected into a single complex constant $c_k^{(i)}$, and the signal model for all $K$ users can now be written as

$$\boldsymbol{y}_i = \sum_{k=1}^{K} \left( c_k^{(i-1)} \boldsymbol{u}_k^r + c_k^{(i)} \boldsymbol{u}_k^l \right) + \eta_i = \boldsymbol{A}\,\boldsymbol{c}_i + \eta_i, \quad (4)$$

where $\boldsymbol{\eta_i} = [\eta_{i,0}, \ldots, \eta_{i,N-1}]^\top \in \mathcal{C}^N$ is a Gaussian random vector and its elements are zero mean with variance $\sigma^2 = \frac{\mathcal{N}_0}{2T_c}$ and are mutually independent. The signal vectors $\boldsymbol{u}_k^r$ and $\boldsymbol{u}_k^l$ depend only on the user's spreading waveform and the associated channel impulse response. In the stationary case, where the code words repeat from bit to bit, the signal vectors do not depend on the time index $i$. However, in the general nonstationary case where code words are non-repetetive and span multiple bits, $\boldsymbol{u}_{k,i}^r$ and $\boldsymbol{u}_{k,i}^l$ vary with $i$. Note that in (4), we have defined $\boldsymbol{c}_i = [c_1^{(i-1)}\, c_1^{(i)} \cdots c_K^{(i-1)}\, c_K^{(i)}]^\top \in \mathcal{C}^{2K}$ and the code matrix

$$\boldsymbol{A} = [\boldsymbol{u}_1^r\, \boldsymbol{u}_1^l \cdots \boldsymbol{u}_K^r\, \boldsymbol{u}_K^l] \in \mathcal{C}^{N \times 2K}\,. \qquad (5)$$

### 2.2 Array Processing

Impinging on an uniformly spaced linear sensor array with $N$ sensors are $K$ narrowband uncorrelated signals centered at frequency $\omega_0$. The sources of the signals are sufficiently far from the sensor array to allow a planar wavefront approximation. Additive noise is present at each sensor and is assumed to be stationary zero mean random processes and independent from sensor to sensor. Then the received signal of the $n$-th sensor $y_n(t)$ is given by

$$y_n(t) = \sum_{k=1}^{K} s_k(t) \mathrm{e}^{-j\omega_0(n-1)\kappa_k} + \eta_n(t)\,, \qquad (6)$$

where

$$\kappa_k = \frac{\Delta}{c} \sin\theta_k$$

and

$$\begin{aligned} \theta_k &= \text{the DOA of the } k\text{–th signal} \\ \Delta &= \text{the spacing between the sensors} \\ c &= \text{the speed of propagation} \\ s_k(t) &= \text{the signal emitted by the } k\text{–th source} \\ \eta_n(t) &= \text{the additive noise at the } n\text{–th sensor} \end{aligned}$$

Again rewriting (6) in matrix notation, we have

$$\boldsymbol{y}_i = \boldsymbol{A}\boldsymbol{s}_i + \boldsymbol{\eta}_i\,, \qquad (7)$$

where

$$\begin{aligned} \boldsymbol{y}_i &= [y_1(t_i), \ldots, y_N(t_i)]^T, \\ \boldsymbol{\eta}_i &= [\eta_1(t_i), \ldots, \eta_N(t_i)]^T, \\ \boldsymbol{s}_i &= [s_1(t_i), \ldots, s_K(t_i)]^T, \end{aligned}$$

and $t_i = i\Delta t$. The array steering matrix $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = [\boldsymbol{a}_1, \ldots, \boldsymbol{a}_K] \in \mathcal{C}^{N \times K}\,, \qquad (8)$$

where $\boldsymbol{a}_k = [1, \ldots, \mathrm{e}^{-j\omega_0(i-1)\kappa_k}, \ldots, \mathrm{e}^{-j\omega_0(N-1)\kappa_k}]^T$

### 2.3 Data Matrix

In both cases the data matrix $\boldsymbol{Y} \in \mathcal{C}^{m \times N}$ is obtained by concatenating the data vectors $\boldsymbol{y}_i$ given by (4) and (7), respectively. Taking $m$ samples we obtain

$$\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{y}_1^T \\ \vdots \\ \boldsymbol{y}_i^T \\ \vdots \\ \boldsymbol{y}_m^T \end{bmatrix} \qquad (9)$$

**Remark 2.1 [Subspace dimensions]** Note that the dimension of the subspaces (equivalent to the dimension of the code matrix/array steering matrix) is different for the CDMA and the array processing case. With $K$ denoting the number of users of the CDMA system and the number of signals impinging on the antenna array we obtain $k = 2K$ and $k = K$, respectively.

## 3 ANGLES BETWEEN THE SUBSPACES

The CS-decomposition of a partitioned orthogonal matrix is a tool for comparing subspaces. In this section we define the CS–decomposition and apply it to the comparison of the subspace given by the code (array steering) matrix $\boldsymbol{A}$ and the signal and noise subspaces, respectively. In particular, we will investigate the angles between the respective subspaces. The following theorem states the CS–decomposition for the special cases (special matrix dimensions) that occur in terms of subspace methods for CDMA delay estimation or DOA estimation.

# DELAY ESTIMATION IN WIRELESS CDMA SYSTEMS USING SUBSPACE ANGLE OPTIMIZATION

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## ABSTRACT

Subspace methods for delay estimation in wireless CDMA systems are presented. The required theory is based on the CS–decomposition which is the basic tool for defining the relation between subspace angles. The unknown parameters are estimated by determing the minimal/maximal subspace angle between the signal/noise subspace and the parameter space. It is shown that using signal and noise subspace angle is equivalent provided that an orthonormal basis for the parameter space is used. Based on this result a subspace algorithm for CDMA delay estimation can be formulated that is identical for signal and noise subspace based estimation. The signal subspace version is essential in order to design efficient (spherical) subspace algorithms. The subspace methods are examined in terms of computational complexity, numerical issues and parallel implementation. The subspace methods for delay estimation in CDMA systems are also compared to the subspace methods for direction–of–arrival estimation in array processing.

## 1 INTRODUCTION

Direct Sequence Code Division Multiple Access (DS–CDMA) systems have been designed in the context of wireless mobile communications. Due to the innate asynchronous nature of the system, the base station receives different users' transmissions at different delays relative to the local clock. To facilitate subsequent detection of the transmitted bits, these delays (or timings) must be accurately estimated. Most of the work done in timing acquisition in CDMA systems involves computationally intensive joint parameter estimation for all the users [1, 2, 3].

Subspace methods for parameter estimation have been used in many signal and array processing applications [4, 5], e.g. spectral estimation, direction–of–arrival (DOA) estimation. Recently these methods have also been applied to timing acquisition in CDMA systems [6, 7]. In contrast to the joint parameter estimation methods [1, 2, 3] the subspace algorithm in [6, 7] results in an elegant and computationally efficient method that reduces the multidimensional problem to one–dimensional problems (see [8] for similar and independent work).

In all the above mentioned applications the received/measured data are arranged in a $L \times N$ matrix $\boldsymbol{Y}$, where $N$ is the spreading gain and $L$ is the number of data samples. Assume that the number of unknown parameters that need to be estimated is $K$. The singular value decomposition (SVD) of the data matrix $\boldsymbol{Y}$ is given by

$$\boldsymbol{Y} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H , \tag{1}$$

where $\boldsymbol{U} \in \mathcal{C}^{L\times N}$ and $\boldsymbol{V} \in \mathcal{C}^{N\times N}$ are orthonormal matrices ($\boldsymbol{U}^T\boldsymbol{U} = \boldsymbol{I}$, $\boldsymbol{V}^T\boldsymbol{V} = \boldsymbol{I}$) and $\boldsymbol{\Sigma} = \mathrm{diag}(\sigma_1, \ldots, \sigma_N)$ is a diagonal matrix containing the singular values $\sigma_i$ of $\boldsymbol{Y}$. We will write $\boldsymbol{\Sigma}(\boldsymbol{Y})$ and $\sigma_i(\boldsymbol{Y})$ if we refer to the singular values of a matrix $\boldsymbol{Y}$ without stating the SVD (1) explicitly. The row space of $\boldsymbol{Y}$ spanned by the orthonormal basis $\boldsymbol{V}$ can be partitioned as $\boldsymbol{V} = [\boldsymbol{V}_S\ \boldsymbol{V}_N]$ where $\boldsymbol{V}_S = \boldsymbol{V}_{1:N,1:k}$ is the signal subspace and $\boldsymbol{V}_N = \boldsymbol{V}_{1:N,k+1:N}$ is the noise subspace. Note, that $k < N$ is assumed with $k = K$ for the antenna array case and $k = 2K$ for the CDMA case.

There are basically two ways to compute the unknown parameters based on the computed subspaces: (1) searching for the otimal projections with respect to the signal subspace (subspace fitting (SF) [9, 10]) or the noise subspace (MUSIC [11]); (2) computing the unknown parameters by a closed form method using the structure of the underlying signal subspace (e.g. shift invariance of the subspace as ESPRIT [12]).

The subspace algorithm in [6, 7] uses the noise subspace (related to MUSIC). However, no search over the whole parameter space is required as in the DOA-estimation case. Instead a closed form expression for determing the locally optimal parameters (timings) within a single chip is derived. Therefore, for determing the globally optimal parameter a search is only required over the set of $N$ locally optimal solutions. Furthermore, the structure of the underlying subspace enables to reduce the $K$–dimensional problem to $K$ one–dimensional problems.

In this paper we analyse the subspace methods for timing acquisition of wireless CDMA systems. Our analysis is based on the CS–decomposition [13, 14] which is essentially a generalized SVD of an orthogonal matrix. The CS–decomposition can be used to define the angle between subspaces. We will apply this definition to the subspace angles present in subspace methods. It is shown that optimizing (maximizing) the angle between the parameter space and the noise subspace is equivalent to optimizing (minimizing) the angle between the parameter space and the signal subspace. However, this is only true if the parameter space is represented by an orthonormal basis. This requires some modifications of the algorithm in [6, 7] which is only applicable for the noise subspace. The resulting signal subspace method enables the design of efficent (spherical) subspace methods for CDMA timing acquistion (e.g [15]).

The paper is organized as follows. In section 2 we introduce the system model for DS–CDMA and array processing using a linear sensor array (for the sake of comparisons given later on). Section 3 gives the main theorem on the relation of the subspace angles, where the CS–decomposition is required for the proof of this theorem. In section 4 the theorem is used for designing signal subspace algorithms for timing acquisition and relating them to the noise subspace algorithms. We also discuss the relation and differences of the CDMA delay estimation problem to the DOA