Updating Directions of Arrival in a GSM-Based SDMA Mobile Radio System

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Abstract

In a space division multiple access (SDMA) mobile radio system, different mobile users use the same spectrum and are separated only by the directions of arrival (DOAs) of their signals at the base station. A critical task is therefore the estimation of the DOAs. When the users move around, the DOAs are time variable, which means that they must be tracked after their initial estimation. Using subspace methods (e.g. ESPRIT, MUSIC) is computationaly expensive and the number of DOAs which can be tracked is limited by the number of antennas.

In this paper an algorithm for updating the DOAs in a GSM-based SDMA mobile radio system is presented. It makes use of the training sequences used in the GSM system and updates the DOAs directly without using the subspaces. The proposed algorithm has low computational complexity and is not limited by the number of antennas.

Monte Carlo simulations show that the tracking behavior of our algorithm is stable and that the error made is small, so that it represents a valuable complement to high-resolution DOA estimation algorithms, which can help decrease the hardware complexity and improve the real-time properties of the SDMA system.

1 Introduction

In a space division multiple access (SDMA) mobile radio system, different users are separated by the directions of arrival (DOAs) of their signals at the base station. Thus, a part of the spectrum can be used not only by one user at a certain time instant, as in conventional time division multiple access (TDMA) or frequency division multiple access (FDMA) systems, but by several users. The condition is that the different signals arriving at the base station are spatially well separated. The main additional task compared to conventional systems that has to be performed in a SDMA system is the estimation of the DOAs.

Recently a series of algorithms for estimating directions of arrival have been published [9, 8]. Different aspects have since been optimized and adapted to the requirements of mobile communications [2, 5]. Before a new connection can be established in a SDMA system, the directions of arrival of the user have to be estimated initially. This can be done using one of the mentioned techniques. Then the traffic channel can begin to operate.

In the traffic channel one has to take into account that the directions of arrival of a user moving at a certain speed will change at a rate depending on this speed. Therefore, the initial estimate has to be updated regularly. It should be avoided to re-run the entire initial estimation algorithm over and over again since this causes a high computational load. Hence, in a SDMA mobile radio system, a computationally efficient real-time updating algorithm would be a valuable complement to the powerful high-resolution techniques mentioned above.

There are several publications dealing with the problem of updating the directions of arrival in the area of sensor array signal processing [1, 3, 6]. The proposed methods essentially update the signal subspaces (e.g. the SVD-updating algorithm). All the algorithms are based on the assumption that the signal subspaces change according to the time variation of the directions of arrival. Our goal is to present an algorithm especially suited for a SDMA mobile radio system based on the existing GSM system. In this special case, we have to take into account that the transmission to a user is discontinuous due to the fact that GSM uses a TDMA modulation scheme. For this reason the basis vectors which span the signal space are completely different in any two successive bursts. Hence, when updating the DOAs in a GSM-based SDMA system, the updating algorithms mentioned above can only be used in a modified way for the case of known data. Here, the training sequences come into play.

Each GSM signal burst contains a known training sequence. The timing offset and the channel coefficients have to be determined for the demodulation of the signal and are therefore available for the DOA estimation algorithm. Since we are dealing with an updating algorithm and the DOAs do not change much from burst to burst, the previous estimates can be used to suppress the other users in the same time slot by performing adaptive spatial filtering. Then the known synchronisation and channel acquisition algorithms of the GSM system can be used. Thus, the training sequence part of the signal can be assumed to be fully known. Under this assumption, the signal subspaces could be updated using one of the known updating algorithms mentioned above. While avoiding performing the complete estimation algorithm again, they still cause a high computational load.

In this paper we propose a simpler and computationally less expensive DOA updating algorithm. Furthermore, for this algorithm the number of DOAs is not limited by the number of antennas. It also makes use of the fact that the training signal is known. It does not work on the signal subspaces but it updates the DOA estimates directly. We will present simulation results showing the ability of the proposed algorithm to track directions of arrival in a GSM–based SDMA mobile radio environment.

2 Data model

2.1 Antenna Array

Our investigations are based on a base station equipped with a uniform linear antenna array with M sensors. There are d wavefronts impinging on the antenna array. The signal sources (the mobile stations) are assumed to be located in the far field of the antenna array. Consequently, the signals are approximated by planar wave fronts. The RF signal of a single wave front received at sensor l can be written as

$$s_l(t) = Re\{u(t-\tau_l)e^{j2\pi f_c(t-\tau_l)}\}. \tag{1}$$

where $u(t)$ is the complex baseband signal, f_c is the carrier frequency and τ_l is the time delay of the signal at sensor l relative to reference sensor 1 (the sensors are numbered in ascending fashion from right to left). An example of a uniform linear array with eight elements is depicted in Fig. 1. Under the assumption of planar wave fronts the time delay τ_l can be computed as follows:

$$\tau_l = (l-1)\frac{\Delta \sin\phi}{c}. \tag{2}$$

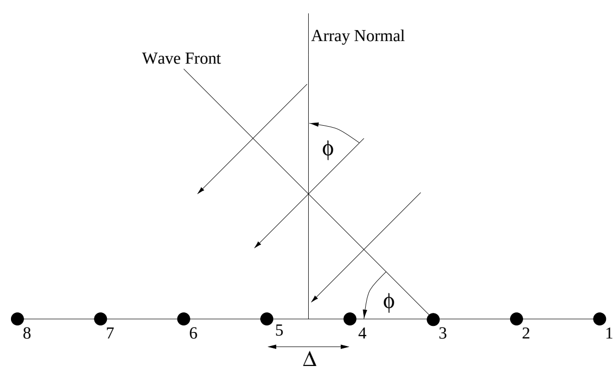

Figure 1: Uniform linear antenna array with eight elements

Here, Δ is the distance of the sensors, ϕ is the angle between the direction of the wave front and the array normal and c is the speed of propagation. Assuming the signals to be narrowband we can introduce the following approximation [8]

$$u(t - \tau_l) \approx u(t), \tag{3}$$

so that the time delay results only in a phase shift and the complex baseband signal can be written as

$$x_l(t) = u(t)e^{-j2\pi f_c \tau_l}. \tag{4}$$

We will now present a matrix formulation for the received signal. The sampled values of the signals received at the sensors of the antenna array constitute the $M \times N$ matrix

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_1{}^T \\ \mathbf{x}_2{}^T \\ \vdots \\ \mathbf{x}_{(M-1)}{}^T \end{bmatrix}, \tag{5}$$

whose row vectors $\mathbf{x}_i{}^T, i = 1, \ldots, M$ contain N sampled values of the signals received at sensor i of the antenna array. We assume that additive Gaussian noise is present at the sensors and that the noise is uncorrelated between different sensors. The $d \times N$ source signal matrix has the following form:

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_1{}^T \\ \mathbf{s}_2{}^T \\ \vdots \\ \mathbf{s}_d{}^T \end{bmatrix}. \tag{6}$$

The row vectors $\mathbf{s}_j{}^T, j = 1, \ldots, d$ represent N sampled values of the d wave fronts impinging on the array. We define the $M \times d$ array steering matrix

$$\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_d], \tag{7}$$

where d is the number of wave fronts. The array steering vectors $\mathbf{a}_i, i = 1, \ldots, d$ have Vandermonde structure:

$$\mathbf{a}_i = \begin{bmatrix} 1 \\ e^{-j2\pi\frac{\Delta \sin \phi_i}{\lambda}} \\ e^{-2j2\pi\frac{\Delta \sin \phi_i}{\lambda}} \\ \vdots \\ e^{-(M-1)j2\pi\frac{\Delta \sin \phi_i}{\lambda}} \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-j\Theta_i} \\ e^{-2j\Theta_i} \\ \vdots \\ e^{-(M-1)j\Theta_i} \end{bmatrix}, \tag{8}$$

where $\Theta_i = 2\pi\frac{\Delta \sin \phi_i}{\lambda}$. Defining $\mathbf{N}$ as a $M \times N$ matrix of uncorrelated Gaussian random variables, we can write the received signal matrix (see (5)) as follows:

$$\mathbf{X} = \mathbf{AS} + \mathbf{N}. \tag{9}$$

2.2 Model of the GSM System

Our algorithm is especially designed for a GSM mobile radio system extended by a uniform linear antenna array with an "intelligent" signal processing unit. The modulation scheme is the one used in existing GSM systems. The bit rate is therefore 270.833 kbit/s; a TDMA frame consists of eight time slots of 576.9 μs resulting in a frame duration of 4.616 ms [7]..

The modulation scheme used is Gaussian Minimum Shift Keying (GMSK) with a 3 db-bandwidth-bit duration product of 0.3 [7]. The premodulation filter has an impulse response given by [7]

$$h_G(t) = \frac{\sqrt{\pi}}{\alpha} e^{-\frac{\pi^2}{\alpha^2}t^2}. \tag{10}$$

The parameter α is related to the bandwidth B by

$$\alpha = \frac{\sqrt{2 \ln 2}}{B}. \tag{11}$$

A burst representing a time slot consists of 156.25 bits with a training sequence of 26 bits in the middle (also called midamble). We assume that at the receiving end the signal is sampled at a rate of 1 sample/bit.

3 Algorithm for estimation of directions of arrival

3.1 Advantages of the proposed updating algorithm

We propose that a signal subspace-based estimation algorithm be used for the initial estimate, e.g. Unitary ESPRIT [5]. The initial estimate is made when a user sets up a connection. A certain number of DOAs can then be estimated and highly resolved, with the limitation that the number of DOAs is always less than the number of antennas M.

This initial estimate is then used by the algorithm we will present in the following. All the DOAs of the users sharing the same time slot can be tracked with the same number of antennas, even if the maximum number of DOAs is initially estimated for each user, such that in the traffic channel the number of DOAs d can be greater than the number of antennas M. The case $d > M$ cannot be handled if any signal subspace-based estimation algorithm is used for updating the DOAs. Moreover, the complexity of the proposed algorithm is low compared to subspace-based algorithms.

3.2 Estimating the array steering matrix

In the following we assume that the signal matrix $\mathbf{S}$ consists of the training sequence part of the burst under investigation only and is therefore fully known (see Section 1). We also assume that $\mathbf{S}$ has full rank and the matrix $\mathbf{S}\mathbf{S}^H$ is therefore invertible.

Under these assumptions, which can realistically be made, we obtain a least squares estimate $\hat{\mathbf{A}}$ for the array steering matrix from (9) by computing

$$\hat{\mathbf{A}} = \mathbf{X}\mathbf{S}^H(\mathbf{S}\mathbf{S}^H)^{-1}. \tag{12}$$

Here and in the following considerations, the "hat" symbol $(\hat{\cdot})$ denotes an estimate. Defining the estimated array steering vectors $\hat{\mathbf{a}}_i, i = 1, \ldots, d$ we can write

$$\hat{\mathbf{A}} = [\hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2, \ldots, \hat{\mathbf{a}}_d]. \tag{13}$$

The inversion of the matrix $\mathbf{S}\mathbf{S}^H$, which is a $d \times d$ matrix (with d being the number of wave fronts) is the most complex computational operation to be performed by the algorithm. The complexity is therefore very low compared to e.g. an ESPRIT-type algorithm which involves the singular value decomposition [4] of the $M \times N$ received signal matrix $\mathbf{X}$.

3.3 Mapping of complex factors to phase rotations

The estimated array steering vectors (see (13))

$$\hat{\mathbf{a}}_i = \begin{bmatrix} 1 \\ e^{-j\widehat{\theta_i}} \\ e^{-j\widehat{2\theta_i}} \\ \vdots \\ e^{-j\widehat{m\theta_i}} \end{bmatrix}, \tag{14}$$

can now be used to determine the estimated phase angle vectors

$$\hat{\varepsilon}_i = \begin{bmatrix} \widehat{\theta_i} \\ \widehat{2\theta_i} \\ \vdots \\ \widehat{m\theta_i} \end{bmatrix}. \tag{15}$$

The problem that arises when mapping the elements $e^{-j\widehat{\theta_i}}, e^{-j\widehat{2\theta_i}}, \ldots, e^{-j\widehat{m\theta_i}}$ of an estimated array steering vector $\hat{\mathbf{a}}_i$ to the phase angles $\widehat{\theta_i}, \widehat{2\theta_i}, \ldots, \widehat{m\theta_i}$ in a vector $\hat{\varepsilon}_i$ is that there are ambiguities. The computation of the phase angle of a complex number can only be performed modulo 2π, so that the true phase angle is not restored if it exceeds 2π.

We can eliminate this ambiguity by iteratively comparing an element of $\hat{\varepsilon}_i$ with the element before and adding 2π to this and all subsequent elements if it is not larger than the element before, that is if $\widehat{k\theta_i} \leq \widehat{(k-1)\theta_i}, 3 < k < m$. This algorithm yields the unambiguous estimate for the phase angle vector

$$\tilde{\varepsilon}_i = \begin{bmatrix} \widetilde{\theta_i} \\ \widetilde{2\theta_i} \\ \vdots \\ \widetilde{m\theta_i} \end{bmatrix}, \tag{16}$$

which will be used to determine the directions of arrival.

3.4 Exploiting the Vandermonde structure

The condition under which the directions of arrival $\phi_i, i = 1, \ldots, d$ can be directly determined from the array steering matrix $\mathbf{A}$ is that the distance Δ between two adjacent sensors of the antenna array fulfills

$$\Delta \leq \frac{\lambda}{2}, \tag{17}$$

where λ is the wavelength of the RF signal. Then it can easily be shown that the phase rotation θ (see (8)) is bounded as follows:

$$0 \leq \theta \leq \pi. \tag{18}$$

When the exact array steering matrix $\mathbf{A}$ is known, the directions of arrival $\phi_i, i = 1, \ldots, d$ can be uniquely determined from the respective second elements $\rho_i = e^{-j\theta_i}$ of the array steering vectors $\mathbf{a}_i$ (see (8)) as follows:

$$\phi_i = \arcsin\left(-\frac{\lambda}{2\pi\Delta}\arctan\left(\frac{Im\{\rho_i\}}{Re\{\rho_i\}}\right)\right). \tag{19}$$

In order to estimate the directions of arrival from the estimated array steering matrix $\hat{\mathbf{A}}$, we use the knowledge that the array steering vectors have Vandermonde structure (see (8)): from the unambiguous estimate of the phase angle vectors $\tilde{\varepsilon}_i$ (see (16))we determine phase angle estimates γ_i by minimizing the norm $\|\mathbf{e}\|$ of the error vector $\mathbf{e}$ in

$$\begin{bmatrix} \gamma_i \\ 2\gamma_i \\ \vdots \\ m\gamma_i \end{bmatrix} = \tilde{\varepsilon}_i + \mathbf{e} \tag{20}$$

in the least squares sense. This operation can be interpreted as least squares line fitting.

Now we can determine estimates of the directions of arrival in a way similar to that followed in (19):

$$\phi_i = \arcsin\left(-\frac{\lambda}{2\pi\Delta}\gamma_i\right). \tag{21}$$

3.5 Complexity of the proposed algorithm

We will give the number of FLOPs to be performed by the proposed algorithm for different scenarios and we will compare these numbers to a standard ESPRIT algorithm [8]. Our calculations are based on [4].

In all cases we assume that the number of samples is $N = 16$ (reliable number of bits in midamble). For the case of two DOAs ($d = 2$) and five antennas ($M = 5$) the proposed algorithm uses 258 FLOPs; this is by a factor of 9.3 better than the ESPRIT algorithm. Augmenting the number of DOAs to nine ($d = 9$) results in 3636 FLOPs for the proposed algorithm and an improvement factor of 7.2. Note that the number of antennas has to be increased to 10 for the ESPRIT algorithm. If 15 DOAs ($d = 15$) are updated using only three antennas ($M = 3$), the proposed algorithm uses 10620 FLOPs. The improvement factor compared to ESPRIT in this case is 10.2, considering that the number of antennas has to be increased to 16 for ESPRIT.

In summary, we can say that the proposed algorithm provides advantages regarding both computational complexity and the number of DOAs that can be updated.

4 Simulations

4.1 Simulation Scenario

The scenario which is the basis for the computer simulations can be seen in Figure 2, which is scaled correctly except for the length of the antenna array. It consists of a base station equipped with a uniform linear antenna array (ULA) with eight elements and of two mobile users that operate in the same time slot of the spectrum. The signals are separated spatially by their directions of arrival. Our simulations are based on the following assumptions. Users 1 and 2 move with constant velocities v_1 and v_2, respectively. There are two incoming wave fronts from each user (referred to as signals 1 to 4 in the following text). One of them is the direct "line of sight" (LOS) connection, whose DOA varies over time. The other is a reflected signal, which is assumed to be reflected at a fixed reflection point. Its DOA therefore remains constant throughout the simulations. The attenuation of the reflected signals relative to the respective line of sight-signals is 3 dB. The DOAs are negative for signals impinging from the left (relative to the array normal) and positive for signals impinging from the right. The initial DOAs of signals 1 to 4 are: $\Phi_1 = -51.34°$ (LOS, time variable), $\Phi_2 = -71.57°$ (constant), $\Phi_3 = 56.31°$ (LOS, time variable), $\Phi_4 = 45.0°$ (constant). The speed of user 1 is 100 km/h, that of user 2 is 70 km/h. The simulations cover 6500 GSM bursts which corresponds to a time duration of 30 seconds. The signal to noise ratio (SNR) in the simulations is 10 dB. Since the synchronization cannot be expected to be exact, a timing error T_{err} chosen randomly from the interval $-\frac{1}{16}T_{bit} < T_{err} < \frac{1}{16}T_{bit}$ is introduced in the simulations, with T_{bit} being the GSM bit duration of 3.692 μs.

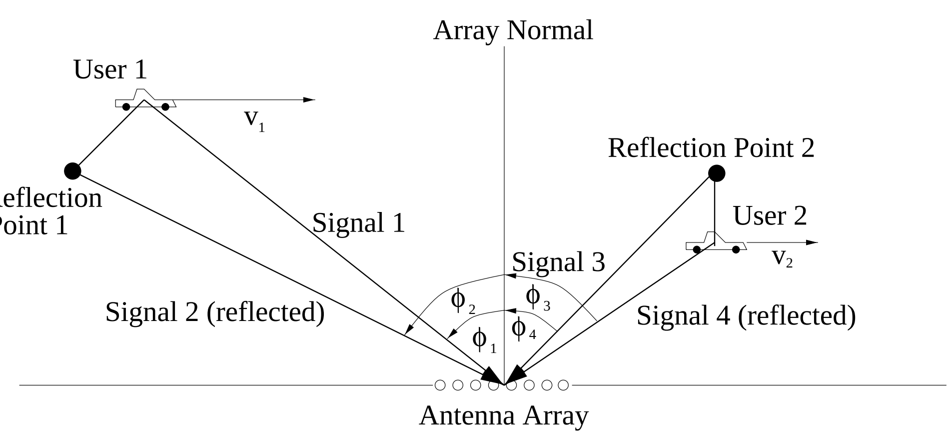

Figure 2: Simulation scenario

4.2 Simulation Results

The results of the computer simulations are depicted in Figures 3 to 10. Figures 3 and 7 show the DOAs of the signals of users 1 and 2, respectively, plotted versus the time in seconds. Figures 4 and 8 show in similar plots how the updating algorithm tracks these DOAs. The absolute deviation of the DOA estimates of the updating algorithm from the true DOAs can be seen in Figures 5, 6, 9 and 10. These deviations are due to the noise and the timing errors. It can be seen that the absolute deviation is in all cases less than 1.5°.

5 Conclusions

We have presented a new algorithm that can be used for updating the time-variable directions of arrival in a GSM-based SDMA system. Our algorithm makes use of the training sequence which is part of every GSM signal burst. It updates the DOAs directly and is not based on subspace estimates. The algorithm exploits the fact that the DOAs vary only by a small amount from time slot to time slot. It yields a method for tracking the DOAs initially estimated by a high-resolution DOA estimation algorithm. We see our direct updating algorithm as a good alternative to signal subspace-based methods due to the high number of DOAs that can be tracked (independent of the number of antennas), its robust tracking behavior and the low computational complexity.

References

[1] C.H. Bischof and G.M. Shroff. On updating signal subspaces. *IEEE Trans. Signal Proc.*, 40:96–105, 1992.

[2] C. Farsakh and J.A. Nossek. Application of SDMA to mobile radio. In *Proc. 5th Int. Symp. on Personal, Indoor and Mobile Radio Comm. (PIMRC 94)*, pages 736–739, The Hague, Netherlands, 1994.

[3] W. Ferzali and J.G. Proakis. Adaptive SVD algorithm for covariance matrix eigenstructure computation. In *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, pages 2615–2618, Toronto (Canada), 1990.

[4] Gene H. Golub and Charles F. van Loan. *Matrix Computations.* The Johns Hopkins University Press, Baltimore and London, 1989.

[5] M. Haardt. *Efficient One-, Two-, and Multidimensional High-Resolution Array Signal Processing.* PhD thesis, Technical University Munich, Institute for Network Theory and Circuit Design, Germany, 1996.

[6] M. Moonen, P. van Dooren, and J. Vandewalle. A singular value decomposition updating algorithm for subspace tracking. *SIAM J. Matrix Anal. Appl.*, 13:1015–1038, 1992.

[7] Theodore S. Rappaport. *Wireless Communications.* Prentice Hall PTR, Upper Saddle River, NJ, 1996.

[8] Richard Roy and Thomas Kailath. ESPRIT - Estimation of signal parameters via rotational invariance techniques. *IEEE Trans. Acoust., Speech and Signal Proc.*, 37(7):984–99, July 1989.

[9] R. O. Schmidt. Multiple emitter location and signal parameter estimation. *IEEE Trans. Antennas and Propagation*, 34(3):276–280, March 1986.

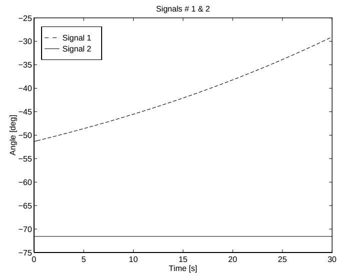

Figure 3: Directions of arrival of signals 1 and 2

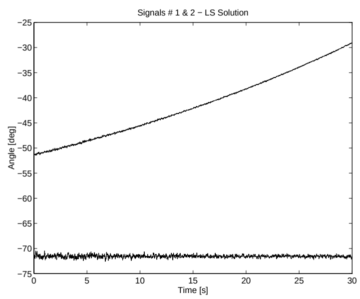

Figure 4: Tracking of the DOAs of signals 1 and 2

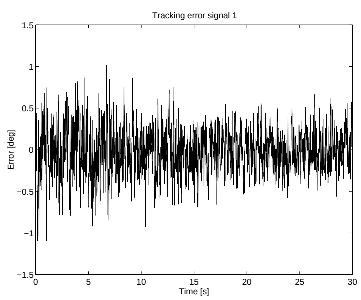

Figure 5: Tracking error signal 1

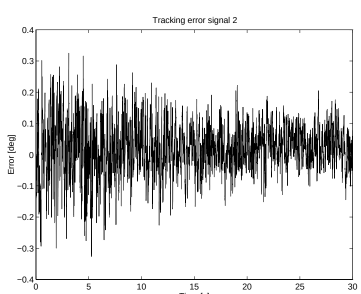

Figure 6: Tracking error signal 2

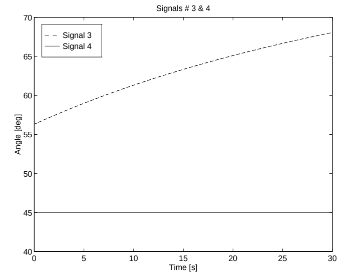

Figure 7: Directions of arrival of signals 3 and 4

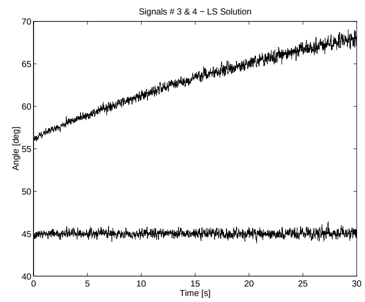

Figure 8: Tracking of the DOAs of signals 3 and 4

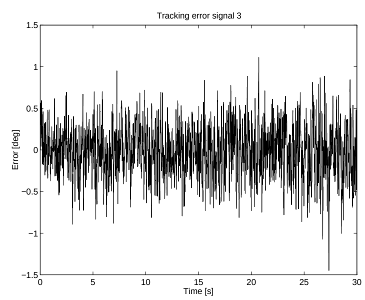

Figure 9: Tracking error signal 3

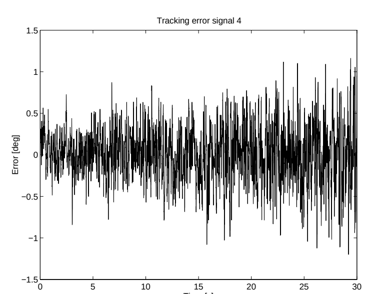

Figure 10: Tracking error signal 4