Ov-OFDM: A Reduced PAPR and Cyclic Prefix Free Multicarrier Transmission System

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***Abstract*—Orthogonal Frequency Division Multiplexing (OFDM) enables high data rate transmissions over frequency selective fading channels. Its beneficial qualities are a low implementation complexity and the possible application of power loading schemes to increase data throughput. Drawbacks of OFDM are the requirement for a cyclic prefix (CP) to avoid interference between OFDM symbols and the high peak to average power ratio (PAPR). The CP causes transmission overhead and hence reduces throughput, while a high PAPR requires the use of less efficient linear amplifiers. The CP overhead can be reduced by increasing the OFDM symbol length, which will, however, increase processing delay and PAPR. In conventional OFDM the symbol length directly determines PAPR and transmission overhead. In this paper we use a CP free OFDM transmission system that allows shortening the OFDM symbol length to reduce the maximum PAPR without transmission overhead trade-off. The proposed system, called Ov-OFDM, is based on overlapping MMSE frequency domain equalization to remove interference between OFDM symbols.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is the method of choice when designing transmission links for high speed wireless communication systems. The reasons are OFDM's beneficial features, like low implementation complexity, high robustness to multipath environments [1] or adaptive modulation/power loading schemes for frequency selective fading channels [2]. Considering this, it is not a surprise that OFDM is applied in most currently used transmission systems, like DVB, DAB, WLAN and DRM. The corresponding multiple access scheme, OFDMA, is used in the WiMax and the 3GPP LTE standard.

However, besides its beneficial qualities, OFDM exhibits two problems, the transmission overhead caused by the cyclic prefix (CP) and the peak to average power ratio (PAPR). The latter will directly influence the energy efficiency of a transceiver, because a high PAPR requires a linear and therefore less energy efficient power amplifier in the transmitter to avoid clipping and distortion. A high PAPR occurs if a large number of subcarriers in an OFDM symbol adds up in-phase. Several approaches were presented to reduce the PAPR, an overview is given in [3]. As the maximum PAPR grows with the number of subcarriers in a symbol, a straightforward way to reduce the PAPR is the reduction of the OFDM symbol length, which will, however, worsen the CP problem.

The CP is inserted between the OFDM symbols to prevent interference due to multipath propagation. This causes a reduction of achievable data throughput, as no information payload can be transferred when transmitting the CP. In the WLAN standard, for example, 20 % of the transmission time is allocated for the CP [4]. A common way to countervail this problem is to increase the OFDM symbol size, which will on the one hand reduce the relative transmission overhead. On the other hand it will, however, increase the FFT implementation complexity, the processing delay, the sensitivity to phase noise and frequency offset, as well as the PAPR [1].

As the CP length is dictated by the delay spread of the wireless channel, the transmission overhead is determined by the OFDM symbol length, which in turn determines the maximum PAPR. In [5] we presented an approach to decouple the OFDM symbol length and the overhead caused by the CP. This is realized by grouping several smaller OFDM symbols into a frame and insert the CP on a frame basis. Prior to data detection the received frame is equalized by a frequency domain equalizer (FDE) [6] to remove interference between the OFDM symbols. This approach was later extended to a scheme called OFDM Time Division Multiplexing (OFDM/TDM) and considered for PAPR reduction in [7]. A similar approach called Multisymbol Encapsulated OFDM (MSE-OFDM) was described in [8].

Though the frame structure can reduce the overhead, the required CP still causes transmission overhead. Furthermore the frame-wise equalization will produce a high processing delay. In [9] we proposed a method to avoid CP insertion while maintaining a block processing structure similar to that of OFDM by using an overlapping equalization scheme. The proposed scheme is based on block-wise equalization, firstly tolerating the interference between the blocks due to the missing CP. The resulting equalization error distribution has a bathtub-like shape, which can be exploited by utilizing only the middle part of each block for further processing, while omitting the outer erroneous parts. Performing this for overlapping data blocks leads to an efficient block-wise equalization with only slightly increased mean square error. While, so far, the main focus regarding overlapping equalization was on single carrier data transmission [10], in this paper the overlapping approach is applied as pre-FFT equalizer for an OFDM system, as suggested in [9]. The resulting overlapping based OFDM system, which is referred to as Ov-OFDM, will be compared to both conventional OFDM and OFDM/TDM in terms of PAPR reduction and computational complexity.

The paper is organized as follows: In Section II a data model for OFDM transmission will be introduced. This model is used to derive conventional OFDM, OFDM/TDM and Ov-OFDM. The PAPR problem is discussed in Section III. Simulation results are used to compare the bit error performance of conventional OFDM and Ov-OFDM in Section IV. An important aspect is the computational complexity, which is considered in Section V. Conclusions are drawn in Section VI.

Throughout the paper lower case and upper case bold letters denote column vectors and matrices respectively.

II. OFDM Transmission Systems

A. Data Model

The model describes the transmission of a modulated data vector $\boldsymbol{d} \in \mathbb{C}^V$ of length V over a time dispersive wireless channel, which is described by its normalized discrete impulse response $\boldsymbol{h} \in \mathbb{C}^L$ of length L. The channel is assumed to be time invariant during the transmission of $\boldsymbol{d}$. The noise vector $\boldsymbol{n}$ is obtained by sampling a white Gaussian noise process with power σ^2. The OFDM symbols are obtained by separating $\boldsymbol{d}$ into segments of length N_S, which are then transformed to time domain by an IFFT (Inverse Fast Fourier Transform) of size N_S. The resulting vector containing the OFDM symbols in time domain is denoted by $\boldsymbol{d}_t$.

The received vector $\boldsymbol{x}_t \in \mathbb{C}^{V+L-1}$ can be computed by convolution of $\boldsymbol{d}_t$ with the impulse response $\boldsymbol{h}$. By using the channel convolution matrix $\boldsymbol{H} \in \mathbb{C}^{(V+L-1)\times(V)}$ the model can be summarized as

$$\boldsymbol{x}_t = \boldsymbol{H}\boldsymbol{d}_t + \boldsymbol{n}. \tag{1}$$

The receiver has to compute an estimate $\hat{\boldsymbol{d}}$ of the transmitted data $\boldsymbol{d}$. Prior to this the interference between successive OFDM symbols has to be removed, which can be realized by channel equalization. It is assumed that estimates of the channel impulse response $\boldsymbol{h}$ and of the noise power σ^2 are available at the receiver. The transmitted data is uncorrelated and has a mean power of $\sigma_d^2 = 1$, therefore the output of the MMSE (Minimum Mean Square Error) equalizer in time domain is given as

$$\hat{\boldsymbol{d}}_t = (\boldsymbol{H}^H\boldsymbol{H} + \sigma^2\boldsymbol{I})^{-1}\boldsymbol{H}^H\boldsymbol{x}_t. \tag{2}$$

After equalization the estimates $\hat{\boldsymbol{d}}$ of the originally transmitted data $\boldsymbol{d}$ can be obtained by symbol-wise application of an FFT of size N_S to $\hat{\boldsymbol{d}}_t$. The direct computation of Equation (2) is hardly feasible for large data vectors, due to the high demand on computing power, storage requirements and large processing delay. By periodically inserting a cyclic prefix of at least $L-1$ samples into $\boldsymbol{d}_t$ prior to transmission the channel convolution matrix $\boldsymbol{H}$ can be split-up into circular submatrices $\boldsymbol{H}_B$ of size $N_B \times N_B$ with $N_B \ll V$. These smaller matrices can be processed independently and reduce the complexity of the MMSE equalizer. In the following the subscript B denotes that we consider blocks of length B.

The circular structure allows the use of FFT-based EVD algorithms (Eigenvalue Decomposition) to compute Equation (2) efficiently. With $\boldsymbol{F}_B$ the Fourier matrix of size B, the EVD of the channel matrix is given as $\boldsymbol{H}_B = \boldsymbol{F}_B^{-1}\boldsymbol{\Lambda}\boldsymbol{F}_B$. The diagonal matrix $\boldsymbol{\Lambda}$ contains the eigenvalues of $\boldsymbol{H}_B$, which can be efficiently computed by Fourier transform of the first column of $\boldsymbol{H}_B$. Applying this the MMSE frequency domain equalizer (MMSE-FDE) can be written as

$$\hat{\boldsymbol{d}}_{t,B} = \boldsymbol{F}_B^{-1}\boldsymbol{\Lambda}_{MMSE}\boldsymbol{F}_B\boldsymbol{x}_{t,B}, \tag{3}$$

where $\boldsymbol{\Lambda}_{MMSE} = (\boldsymbol{\Lambda}^H\boldsymbol{\Lambda} + \sigma^2\boldsymbol{I})^{-1}\boldsymbol{\Lambda}^H$ is a diagonal matrix containing the MMSE equalizer taps in frequency domain.

B. Conventional OFDM

In a conventional OFDM system a CP is added to each symbol, so block size N_B and symbol size N_S are identical. In this case the IFFT of the equalizer and the FFT for data estimation compensate each other, i.e. $\boldsymbol{F}_S\boldsymbol{F}_B^{-1} = \boldsymbol{I}$. This simplifies Equation (3) to

$$\hat{\boldsymbol{d}}_{OFDM} = \boldsymbol{\Lambda}_{MMSE}\boldsymbol{F}_B\boldsymbol{x}_{t,B}. \tag{4}$$

Note here, that all derivations are made using the MMSE equalizer approach. Alternatively least squares equalization or maximum ratio combining (MRC, i.e. multiplication by the conjugate of $\boldsymbol{\Lambda}$ [1]) can be applied. The conventional OFDM transmission system and the resulting transmission signal structure are depicted in Figure 1(a).

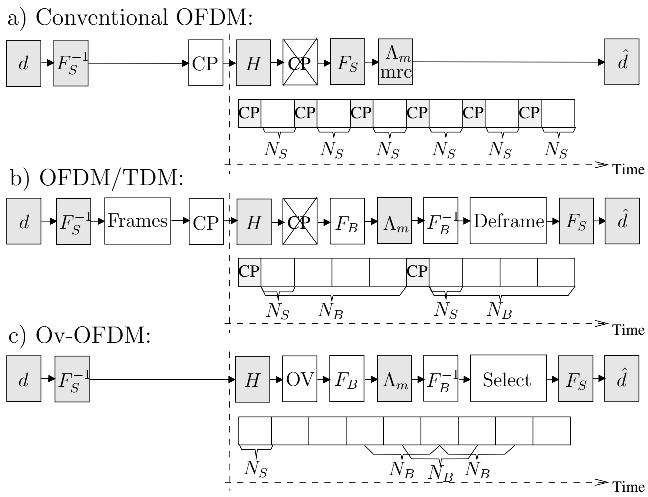

Fig. 1. Overview of the considered OFDM systems.

C. OFDM/TDM and MSE-OFDM

The approaches in [5], [7] and [8], which are for simplicity referred to as OFDM/TDM in the following, are all based on the idea that several OFDM symbols of size N_S are grouped into a frame or block of length N_B. The CP is then added to each frame. This decouples the block size N_B and the OFDM symbol size N_S. As generally $N_B \gg N_S$ the transmission overhead introduced by the CP is reduced. Using the frequency domain equalizer described in Equation (3) the equalization is performed on a frame/block basis , which can result in a large processing delay for increasing N_B. The data in frequency domain is obtained by applying FFTs of size N_S after equalization. The OFDM/TDM transmission system and its corresponding frame structure are depicted in Figure 1(b).

D. Ov-OFDM

In conventional OFDM and OFDM/TDM the block structure is obtained by inserting CPs between consecutive symbols/frames. What does happen if we omit the guard periods and still perform block-wise equalization in the receiver, i.e. estimate the data block $\hat{\boldsymbol{d}}_{t,B}$ using a received data block $\boldsymbol{x}_{t,B}$ and the matrix $\boldsymbol{H}_B$ (cf. Figure 2)?

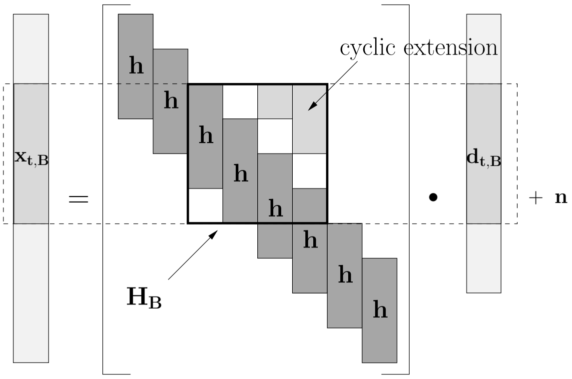

Fig. 2. Block construction for overlapping equalization.

Due to missing CP $\boldsymbol{x}_{t,B}$ also depends on columns of $\boldsymbol{H}$ that are not included in $\boldsymbol{H}_B$. Using $\boldsymbol{H}_B$ for equalization will consequently result in interference that will corrupt the estimated data. However, due to the finite channel length we can expect that the distorting influence of the neighboring blocks is more significant in the border parts of the equalized blocks [9]. To illustrate this, the ensemble-averaged equalization error for three neighboring blocks is depicted in Figure 3(a).

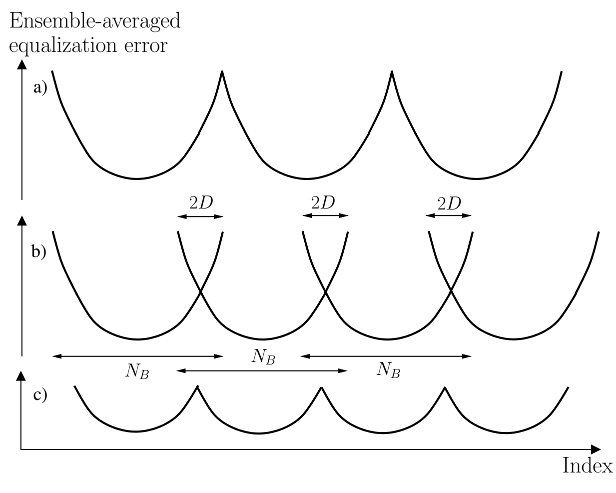

Fig. 3. Error distribution for overlapping equalization.

This bathtub like error distribution can be exploited by using overlapping data blocks instead of neighboring blocks, i.e. a block with elements $n, ..., n + N_B$ is followed by a block $n + N_B - 2D, ..., n + 2N_B - 2D$, as depicted in Figure 3(b). Here $2D$ describes the length of the overlapping parts. The equalization error can then be reduced by omitting the overlapping, more erroneous outer parts of each block and selecting the middle parts for further processing. The resulting ensemble-averaged equalization error is depicted in Figure 3(c).

To allow the use of efficient FFT based EVD algorithms for MMSE equalization (Equation (3)), the overlapping block matrices are cyclically extended (cf. Figure 2). This also means that the underlying signal processing structure is similar to known CP-OFDM and FDE systems.

The equalized signal is transformed to frequency domain by multiple FFTs of size N_S. By applying the FFTs the remaining equalization error in time domain (cf. Figure 3(c)) will be spread over all subcarriers in frequency domain. This leads to an almost uniform error distribution in frequency domain, with a mean power that is below noise level over a wide range of SNR. The impact of the equalization error to the bit error rate performance is clarified by simulations in Section IV. The resulting Ov-OFDM system and its transmission signal structure are depicted in Figure 1(c).

Note that due to CP avoidance, in contrary to OFDM/TDM, the block size N_B can be chosen arbitrarily without affecting the throughput of the system. However, the chosen block size and overlapping length will influence both the computational complexity and the remaining equalization error.

III. Peak to Average Power Ratio

Compared to other transmission techniques, multicarrier systems suffer from a high PAPR, which makes linear amplifiers necessary in the transmitter. Due to their reduced efficiency this is a serious drawback for battery driven devices. For the considered systems the PAPR is defined as the peak power of an OFDM symbol divided by its average power, i.e.

$$PAPR = \frac{\max |\boldsymbol{d}_{t,S}|^2}{E\{|\boldsymbol{d}_{t,S}|^2\}}. \tag{5}$$

Here $\boldsymbol{d}_{t,S}$ describes an OFDM symbol of length N_S in time domain and $E\{\}$ is the ensemble average. The maximum PAPR results from in-phase addition of all subcarriers in an OFDM symbol, i.e. the maximum PAPR grows with the number of subcarriers N_S in a symbol. However, for large N_S the occurrence of the maximum PAPR has a quite small probability.

For a sufficiently large number of subcarriers real and imaginary values of the OFDM signal in time domain can be considered as Gaussian distributed. This allows to describe the probability of the PAPR exceeding a specific limit $PAPR_0$ [1] as

$$P(PAPR > PAPR_0) = 1 - (1 - e^{-PAPR_0})^{N_S}. \tag{6}$$

To avoid inaccuracies due to small N_S the PAPR for different symbol lengths were computed by simulations. The resulting CCDFs [3] are given in Figure 4.

For bit error rate simulations in Section IV we assume a symbol size of $N_S = 32$ for OFDM/TDM and Ov-OFDM.

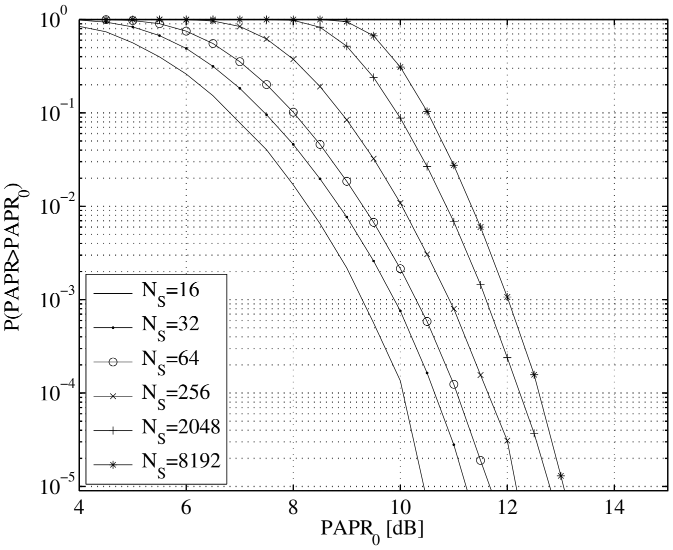

Fig. 4. CCDF of peak to average power ratio for different FFT sizes.

Compared to conventional OFDM with an assumed symbol size of $N_S = 256$ a PAPR reduction of about $1dB$ can be achieved for a clipping probability of 10^{-5}. When considering $N_S = 16$ for Ov-OFDM and $N_S = 8192$ for conventional OFDM the gain increases to $2.5dB$. The PAPR reduction capability of Ov-OFDM is identical to that of OFDM/TDM, however, the latter still requires a CP, which reduces the achievable throughput.

IV. Simulation Results

The simulation results were generated using a symbol spaced multi path fading channel model. A detailed description of the channel model can be found in [11]. The channel length is set to $L = 17$ taps with decaying power distribution. The channel coefficients are assumed to be time invariant during the transmission of one data vector. For all considered systems channel estimation is performed using a training sequence of 128 samples that is transmitted at the beginning of each data vector. The subcarriers are QPSK (Quadrature Phase Shift Keying) modulated.

The frequency selective behavior of the channel can lead to channel zeros at specific subcarrier locations. To restore affected data symbols and to use the full frequency diversity of the OFDM system, channel coding has to be employed. For simulation a convolutional code with constraint length $K = 7$ and code rate $R = 1/2$ was applied. Decoding was performed by a Viterbi decoder using soft inputs [12].

For the symbol spaced simulation system the signal to noise ratio can be expressed by

$$\mathrm{SNR}(dB) = \frac{E_b}{N_0}(dB) + 10\log_{10}(\eta), \tag{7}$$

where η is the spectral efficiency in bits/s/Hz [1][8]. CP insertion will reduce the spectral efficiency, i.e. one OFDM symbol of length $N_{CP} + N_S$ contains only N_S modulated data symbols. With M the modulation order and R the rate of the used channel code, the spectral efficiency is given as

$$\eta = \frac{N_S}{N_S + N_{CP}} R \log_2 M. \tag{8}$$

The relation in Equation (7) shows a loss of SNR due to the CP [1], which will directly influence the achievable bit error rate (BER) regarding E_b/N_0. The SNR loss for a rate $R = 1/2$ code and QPSK modulation is given in Table I for different symbol lengths. The parameter *Size* corresponds to N_S in case of conventional OFDM and N_B in case of OFDM/TDM. Overhead for channel estimation is not considered here as it is identical for all systems.

TABLE I
SNR LOSS DUE TO CP FOR $N_{CP} = 16$ FOR CONVENTIONAL OFDM AND OFDM/TDM. NO CP IS REQUIRED FOR OV-OFDM.

Size	32	64	256	1024	8192
SNR loss/dB	1.7609	0.9691	0.2633	0.0673	0.0085

The SNR loss is also visible in the simulation results, given in Figure 5. While the theoretical loss for a symbol length of $N_S = 256$ is about $1/4$ dB, the simulation results show a slightly smaller gap for the lower range of E_b/N_0. This is caused by the differences in the used equalizers, i.e. maximum ratio combining for conventional OFDM and hybrid MMSE/MRC for Ov-OFDM.

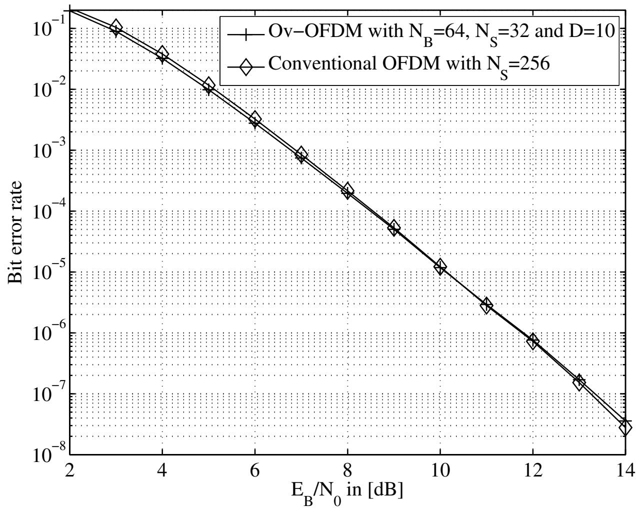

Fig. 5. BER comparison for conventional OFDM and Ov-OFDM.

For larger E_b/N_0 values the gap between the two considered systems reduces, as the remaining equalization error becomes more significant. For a E_b/N_0 larger than $11dB$ (corresponds to a BER of $3 \cdot 10^{-6}$) the remaining errors become dominant compared to the added Gaussian noise. However, in the BER region that is significant for most transmission systems, Ov-OFDM performs comparable or even better than conventional OFDM, while avoiding the overhead caused by the CP and reducing the processing delay. Note that the BER of Ov-OFDM can be further improved by increasing N_B and D, which will, however, also increase the computational complexity.

V. Computational Complexity Considerations

The complexity of all three considered OFDM transmission systems is dominated by the FFT and the inverse FFT respectively. With $\frac{N}{2}\log_2 N$ the number of complex multiplications required to compute one FFT, the total number of multiplications per data symbol necessary for a conventional OFDM system is given by

$$M_{OFDM} = 1 + \frac{1}{2}\log_2 N_S. \tag{9}$$

OFDM/TDM requires frequency domain equalization prior to data detection. This sums up to

$$M_{TDM} = 1 + \log_2 N_B + \frac{1}{2}\log_2 N_S. \tag{10}$$

Due to the overlap in the Ov-OFDM system, more data blocks have to be processed by the equalizer, so the total number of multiplications is given by

$$M_{Ov-OFDM} = \frac{N_B}{N_B - 2D}(\log_2 N_B + 1) + \frac{1}{2}\log_2 N_S. \tag{11}$$

Table II shows the number of required multiplications for different block sizes. As there is no SNR loss due to CP in Ov-OFDM, the block size is fixed to $N_B = 64$. The symbol length of OFDM/TDM and Ov-OFDM is set to $N_S = 32$, for conventional OFDM $N_S = N_B$ holds.

TABLE II
Multiplications per data symbol for OFDM systems with different block size. For Ov-OFDM the block size is fixed to $N_B = 64$ with an overlapping length of $D = 10$.

N_B	**Conv. OFDM**	**OFDM/TDM**	**Ov-OFDM**
32	3.5	8.5	12.68
64	4.0	9.5	12.68
256	5.0	11.5	12.68
1024	6.0	13.5	12.68
8192	7.5	16.5	12.68

While the complexity of Ov-OFDM is fixed, the number of multiplications required for conventional OFDM depends on the symbol size. For small symbols the difference in complexity is large, however, the overhead caused by the CP yields a significant SNR loss (cf. Table I). An increased symbol size will on the one hand reduce the SNR loss of conventional OFDM and also the complexity gap compared to Ov-OFDM. On the other hand, the larger OFDM symbols have a higher PAPR, an increased sensitivity to phase noise and frequency offset and a larger processing delay, which is a major drawback for mobile communication systems.

Similar relations can be found between OFDM/TDM and Ov-OFDM. However, as the complexity of OFDM/TDM is more than two times higher than that of conventional OFDM, there is only a small gap in complexity for small block lengths compared to Ov-OFDM. For larger blocks the complexity of Ov-OFDM will be even lower, while avoiding the CP completely.

VI. Conclusions

The proposed Ov-OFDM transmission scheme is based on overlapping equalization to remove interference between OFDM symbols. This allows to omit the cyclic prefix between OFDM symbols and consequently increases the spectral efficiency of the transmission system. Without cyclic prefix, the trade-off between OFDM symbol size and overhead in terms of ratio between symbol size and CP length is no longer required. This enables the use of very small OFDM symbols, which in turn have a lower peak to average power ratio. Simulations show that the BER degradation due to remaining equalization error is insignificant over a wide range of E_b/N_0. The moderately increased computational complexity is tolerable, especially when considering uplink scenarios. While in uplink case the increased complexity only affects the base station, the complexity of the mobile's transmitter is even reduced, as instead of one huge FFT several small FFTs have to be computed. Combined with the reduced PAPR and the avoidance of transmitting a CP, the energy consumption of the mobile transmitter can be greatly reduced.

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