Improving OFDM Data Estimation by Overlapping Based Cyclic Prefix Reuse

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Abstract— **Orthogonal Frequency Division Multiplexing (OFDM) enables high throughput data transmissions over frequency selective fading channels. Using a cyclic prefix the transmission channel can be described by circular convolution, which allows low complexity FFT based receiver implementations. Generally the CP is discarded at the receiver prior to data estimation, i.e. the redundancy contained in the CP is lost. This redundancy, however, can be used to improve data estimation and hence reduce the bit error rate of the transmission system. This paper considers overlapping frequency domain equalization to exploit CP redundancy for improved OFDM data estimation.**

I. INTRODUCTION

ORTHOGONAL Frequency Division Multiplexing (OFDM) is widely used in high speed wireless communication systems. The main reasons are OFDM's low receiver implementation complexity and its high robustness to frequency selective channel effects [1]. Considering this, it is not a surprise that OFDM is applied in most currently used transmission systems, like DVB, DAB, WLAN and DRM. The corresponding multiple access scheme, OFDMA, is used in WiMax and 3GPP LTE.

An important aspect of OFDM is the cyclic prefix: On the one hand it works as a guard period between OFDM symbols preventing inter symbol interference (ISI). On the other hand it transforms the linear channel convolution into a circular convolution enabling low complexity FFT based receiver implementations.

However, the CP will reduce the achievable data throughput, as no information payload can be transferred when transmitting the CP. In the WLAN standard, for example, 20 % of the transmission time is allocated for the CP [2]. Furthermore transmission energy is wasted as the CP is discarded at the receiver and does not contribute to data estimation.

One approach to tackle this problem is the use of channel equalizers in the receiver instead of CPs to remove ISI between OFDM symbols. In [3] we proposed an overlapping frequency domain equalizer (FDE) that can be used as pre-FFT equalizer in a CP free OFDM transmission systems, referred to as Ov-OFDM [4].

The avoidance of CP, however, requires modifications at the transmitter, which cannot be realized when dealing with existing transmission standards. In these cases the additional information in the CP can be used to improve the quality of the data estimation, instead of discarding it at the receiver. Corresponding approaches can be divided into two categories: Methods that exploit only parts of the CP that are free of interference, i.e. the channel length is shorter than the CP ([5] [6]), and methods that can use the full CP to improve data estimation [7]. The latter can achieve maximum SNR improvement, however, they also require more complex ISI cancellation prior to averaging the redundant data. In [7] an initial data estimation is obtained by conventional OFDM demodulation, which is then used for ISI cancellation, followed by a second data estimation step using the CP redundancy. However, due to its structure this approach is prone to error propagation.

In this paper the overlapping FDE described in [3] is used to remove ISI to improve data estimation by exploiting redundancy using the full CP. Two approaches will be described: The first one, referred to as overlapping CP reuse, is realized by completely equalizing the received CP-OFDM signal without discarding the CP. This yields two independent estimates for the CP data block, one in front and one at the end of each symbol. By averaging these estimates, which are affected by uncorrelated noise, the signal to noise ratio (SNR) can be increased.

The second one, referred to as overlay CP reuse, equalizes solely the CP parts in front and at the end of each symbol. These can be used to compute a correction factor that will be added to the frequency domain estimates obtained by conventional OFDM to improve data estimation quality.

The paper is organized as follows: In Section II a data model for OFDM transmission will be introduced. A short introduction to overlapping FDE is given in Section III. The application of overlapping FDE for CP reuse is described in Section IV. Simulation results are used to compare the bit error rate (BER) of conventional OFDM and OFDM with CP reuse in Section V. An important aspect is the computational complexity, which is considered in Section VI. Conclusions are drawn in Section VII.

II. OFDM SYSTEM MODEL

The model describes the transmission of a modulated data vector $\boldsymbol{d} \in \mathbb{C}^V$ of length V over a time dispersive wireless channel, which is described by its normalized discrete impulse response $\boldsymbol{h} \in \mathbb{C}^L$ of length L. The channel is assumed to be time invariant during the transmission of $\boldsymbol{d}$. The noise vector $\boldsymbol{n}$ is obtained by sampling a white Gaussian noise process with power σ^2. The OFDM symbols are obtained by separating $\boldsymbol{d}$ into segments of length N_s, which are then transformed to time domain by an IFFT (Inverse Fast Fourier Transform) of size N_s. The resulting vector containing the OFDM symbols in time domain is denoted by $\boldsymbol{d}_t$.

The received vector $\boldsymbol{x}_t \in \mathbb{C}^{V+L-1}$ can be computed by convolution of $\boldsymbol{d}_t$ with $\boldsymbol{h}$. By using the channel convolution matrix $\boldsymbol{H} \in \mathbb{C}^{(V+L-1)\times(V)}$ the model can be summarized in

$$\boldsymbol{x}_t = \boldsymbol{H}\boldsymbol{d}_t + \boldsymbol{n}. \tag{1}$$

The receiver has to compute an estimate $\hat{\boldsymbol{d}}$ of the transmitted data $\boldsymbol{d}$. A cyclic prefix of at least $N_{cp} = L - 1$ samples periodically inserted between consecutive OFDM symbols avoids interference and splits the matrix $\boldsymbol{H}$ up into smaller circular submatrices $\boldsymbol{H}_s$ of size $N_s \times N_s$. With $\boldsymbol{F}_s$ the Fourier matrix of size N_s these submatrices can be represented by their EVD (Eigenvalue Decomposition) as $\boldsymbol{H}_s = \boldsymbol{F}_s^{-1}\boldsymbol{\Lambda}\boldsymbol{F}_s$. The diagonal matrix $\boldsymbol{\Lambda}$ contains the eigenvalues of $\boldsymbol{H}_s$. With this the received signal for one OFDM symbol is given as

$$\boldsymbol{x}_{t,s} = \boldsymbol{F}_s^{-1}\boldsymbol{\Lambda}\boldsymbol{d}_{t,s} + \boldsymbol{n}. \tag{2}$$

Demodulation is performed by a single Fourier transform and multiplication by a diagonal matrix $\boldsymbol{\Lambda}_m$:

$$\hat{\boldsymbol{d}} = \boldsymbol{\Lambda}_m\boldsymbol{F}_s\boldsymbol{x}_{t,s} \tag{3}$$

The matrix $\boldsymbol{\Lambda}_m = (\boldsymbol{\Lambda}^H\boldsymbol{\Lambda} + \sigma^2\boldsymbol{I})^{-1}\boldsymbol{\Lambda}^H$ describes the minimum mean square error (MMSE) equalizer in frequency domain. Note that all derivations can be simply modified to support least squares (LS) estimation or maximum ratio combining (MRC) [1].

III. OVERLAPPING FDE

Instead of using CPs the interference between consecutive OFDM symbols could be removed using an equalizer prior to demodulation. Given the received signal in Eq.(1), the MMSE equalizer in time domain is given as

$$\hat{\boldsymbol{d}}_t = (\boldsymbol{H}^H\boldsymbol{H} + \sigma^2\boldsymbol{I})^{-1}\boldsymbol{H}^H\boldsymbol{x}_t. \tag{4}$$

After equalization the estimates $\hat{\boldsymbol{d}}$ of the originally transmitted data $\boldsymbol{d}$ can be obtained by symbol-wise application of an FFT of size N_s to $\hat{\boldsymbol{d}}_t$.

Due to missing CPs an efficient computation of Eq.(4) using independent block submatrices is not straightforward. However, what happens if we still perform block-wise equalization in the receiver, i.e. equalize a data block $\boldsymbol{x}_{t,B}$ of size N_B using the matrix $\boldsymbol{H}_B$ like shown in Figure 1?

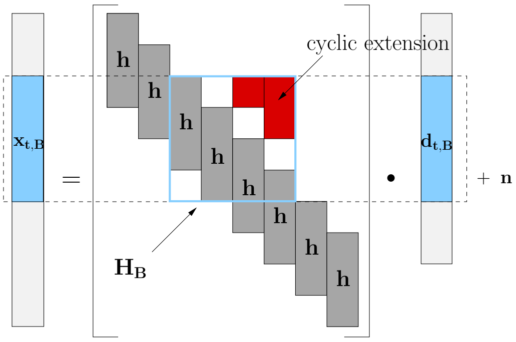

Fig. 1. Block construction for overlapping equalization.

This consequently results in interference that will corrupt the estimated data. However, due to the finite channel length we can expect that the distorting influence of the neighboring blocks is more significant in the border parts of the equalized blocks [3]. To illustrate this, the ensemble-averaged equalization error for three neighboring blocks is depicted in Figure 2(a).

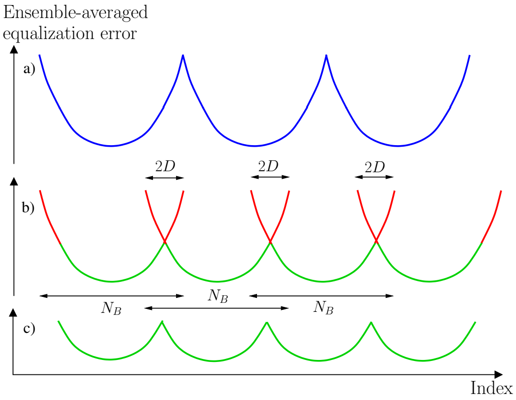

Fig. 2. Error distribution for overlapping equalization.

This bathtub like error distribution can be exploited by using overlapping data blocks instead of neighboring blocks, i.e. a block with elements $n, ..., n + N_B$ is followed by a block $n + N_B - 2D, ..., n + 2N_B - 2D$, as depicted in Figure 2 (b). Here $2D$ describes the length of the overlapping parts. The equalization error can then be reduced by omitting the overlapping, more erroneous outer parts of each block and selecting the middle parts for further processing. The resulting ensemble-averaged equalization error of the output sequence is depicted in Figure 2(c).

To allow the use of efficient FFT based EVD algorithms for MMSE equalization the overlapping block matrices are cyclically extended, which results in signal processing structures similar to that of conventional CP-OFDM systems. The MMSE-FDE for one block is then given as

$$\hat{\boldsymbol{d}}_{t,B} = \boldsymbol{F}_B^{-1}\boldsymbol{\Lambda}_m\boldsymbol{F}_B\boldsymbol{x}_{t,B}. \tag{5}$$

After equalization the middle parts of $\hat{\boldsymbol{d}}_{t,B}$ are selected and combined to obtain the entire equalized output sequence $\hat{\boldsymbol{d}}_t$.

IV. CP REUSE

Instead of discarding it at the receiver, the CP can be used to improve data estimation by averaging independent estimates of the CP itself and the corresponding data part at the end of the OFDM symbol. These can be obtained using the overlapping FDE described in the previous section.

A. Overlapping CP Reuse

The received sequence including the CPs is partitioned into overlapping parts $\boldsymbol{x}_{t,B}$ of length N_B. These parts are then equalized corresponding to Eq.(5). The combined output sequence now contains two independent estimates of the CP data, which can be simply averaged to improve data estimation:

$$\hat{\boldsymbol{d}} = \boldsymbol{F}_s \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_{s-c} & \boldsymbol{0} \\ \frac{1}{2}\boldsymbol{I}_c & \boldsymbol{0} & \frac{1}{2}\boldsymbol{I}_c \end{bmatrix} \hat{\boldsymbol{d}}_{t,s+c}. \tag{6}$$

Here $\hat{\boldsymbol{d}}_{t,s+c}$ denotes one equalized OFDM symbol plus CP taken from $\hat{\boldsymbol{d}}_t$ and $\boldsymbol{I}_c$ is the unity matrix of size N_{cp}. After averaging OFDM demodulation is performed by symbol-wise Fourier transform. The resulting receiver structure and overlapping scheme are depicted in Figure 3(b). For comparison the conventional OFDM system is given in Figure 3(a).

B. Overlay CP Reuse

Using overlapping FDE one OFDM symbol will be partitioned into several parts of length N_B with $N_B < N_s$. This means that remaining equalization errors in the border parts of all these blocks can degrade the quality of data estimation. It would be desirable to apply Eq.(5) only for parts that really have to be equalized, i.e. use normal OFDM data estimation for the main part of the symbol and apply equalization only for the CP parts. The data inside the OFDM symbol can be equalized by simple cyclic prefix based FDE. This leads to $\hat{\boldsymbol{d}}_{t,o} = \boldsymbol{F}_s^{-1}\Lambda_m\boldsymbol{F}_s\boldsymbol{x}_{t,s}$. The equalized CP can be directly obtained from Eq.(5). With this the averaging can be described as

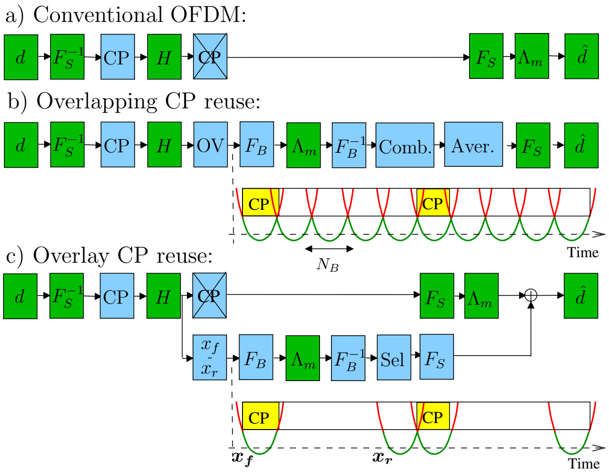

Fig. 3. Block diagram of different OFDM receivers.

$$\hat{\boldsymbol{d}} = \boldsymbol{F}_s \left(\begin{bmatrix} \boldsymbol{I}_{s-c} & \boldsymbol{0} \\ \boldsymbol{0} & \frac{1}{2}\boldsymbol{I}_c \end{bmatrix} \hat{\boldsymbol{d}}_{t,o} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \frac{1}{2}\boldsymbol{I}_c \end{bmatrix} \hat{\boldsymbol{d}}_{t,s} \right).$$

The CP-FDE equalization can be avoided to reduce computational complexity. In a first step the previous equation will be rearranged to

$$\begin{aligned} \hat{\boldsymbol{d}} &= \Lambda_m\boldsymbol{F}_s\boldsymbol{x}_{t,s} \\ &+ \boldsymbol{F}_s \left(\begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & -\frac{1}{2}\boldsymbol{I}_c \end{bmatrix} \hat{\boldsymbol{d}}_{t,o} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \frac{1}{2}\boldsymbol{I}_c \end{bmatrix} \hat{\boldsymbol{d}}_{t,s} \right). \end{aligned} \tag{7}$$

The first part describes conventional OFDM estimation, the second part can be seen as a correction factor, referred to as overlay. It is obvious that only the equalized CP part in the rear of $\hat{\boldsymbol{d}}_{t,o}$ is required, which can be simply obtained from Eq.(5). Considering this the overlay CP approach can be described as

$$\begin{aligned} \hat{\boldsymbol{d}} &= \Lambda_m\boldsymbol{F}_s\boldsymbol{x}_{t,s} \\ &+ \boldsymbol{F}_s \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \frac{1}{2}\boldsymbol{I}_c \end{bmatrix} \boldsymbol{P}\boldsymbol{F}_B^{-1}\Lambda_m\boldsymbol{F}_B\left(\boldsymbol{x}_{t,f} - \boldsymbol{x}_{t,r}\right). \end{aligned} \tag{8}$$

The vectors $\boldsymbol{x}_{t,f}$ and $\boldsymbol{x}_{t,r}$ are suitable inputs for the equalizer to obtain the equalized CP parts in front and at the end of each OFDM symbol. The matrix $\boldsymbol{P}$ is a permutation matrix that shifts the usable middle part of each equalized block (which contains the equalized CP) to the end of the symbol. Note that for this approach the overlap-select-combine procedure of the overlapping FDE simplifies to a single select and shift operation. The resulting receiver structure and the used parts of the received sequence are depicted in Figure 3(c). Note that the upper signal path shows a conventional OFDM receiver and the lower path computes the overlay. Depending on the actual SNR the overlay path can be simply activated to improve the data estimation or deactivated to reduce computational effort.

V. SIMULATION RESULTS

The simulation results were generated using a symbol spaced multi path fading channel model [8]. Position and average power of the channel taps are chosen according to ETSI channel P6 [9]. To model single frequency network (SFN) behavior these taps are repeated with lower power at position $N_{cp}/2$, as shown in Figure 4 for $N_{cp} = 256$. The channel coefficients are assumed to be time invariant during the transmission of one data vector and are known at the receiver. The subcarriers are QPSK (Quadrature Phase Shift Keying) modulated. Prior to transmission the data is encoded using a convolutional code with constraint length $K = 7$ and code rate $R = 1/2$. Decoding is performed by a Viterbi decoder using soft inputs. The parameters are chosen as $N_B = 4N_{cp}$, $D = 0.75N_{cp}$ for the overlapping CP reuse approach and $N_B = 2N_{cp}$, $D = 0.5N_{cp}$ for the overlay approach. Note that block size N_B and overlapping length D are design parameters that allow a trade-off between equalization error and computational complexity.

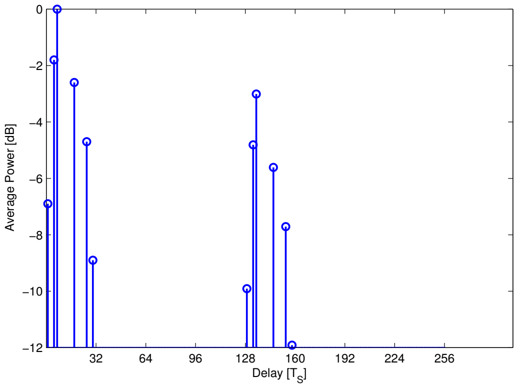

Fig. 4. Average Power of Channel Taps for $N_{cp} = 256$.

Exploiting the CP redundancy will increase the SNR of the received signal. When averaging the CP part in front and at the end of each symbol the signal power remains unaltered, while the noise power will be reduced. Under the assumption of perfect ISI cancellation the improved SNR can be calculated as the ratio between signal power P_S and mean noise power $\overline{P}_N$, i.e.

$$\begin{aligned}\mathrm{SNR}_i(dB) =& 10\log_{10}(\frac{P_S}{\overline{P}_N}) \\ =& 10\log_{10}(\frac{P_S}{P_N\frac{N_s-N_{cp}}{N_s}+\frac{1}{2}P_N\frac{N_{cp}}{N_s}}) \\ =& \mathrm{SNR}(dB)+10\log_{10}(\frac{1}{1-\frac{1}{2}\frac{N_{cp}}{N_s}}).\end{aligned} \tag{9}$$

The SNR improvement compared to conventional OFDM for different ratios N_{cp}/N_s is given in Table I. Assuming a ratio $N_{cp}/N_s = 1/4$ (e.g. WLAN or DVB-T), an SNR improvement of more than $1/2$ dB can be achieved, which is also visible in the simulation results given in Figure 5 for a wide range of E_b/N_0.

TABLE I

SNR GAIN FOR DIFFERENT RATIOS N_{cp}/N_s.

N_{CP}/N_S	1/4	1/8	1/16	1/32
SNR gain/dB	0.580	0.280	0.138	0.068

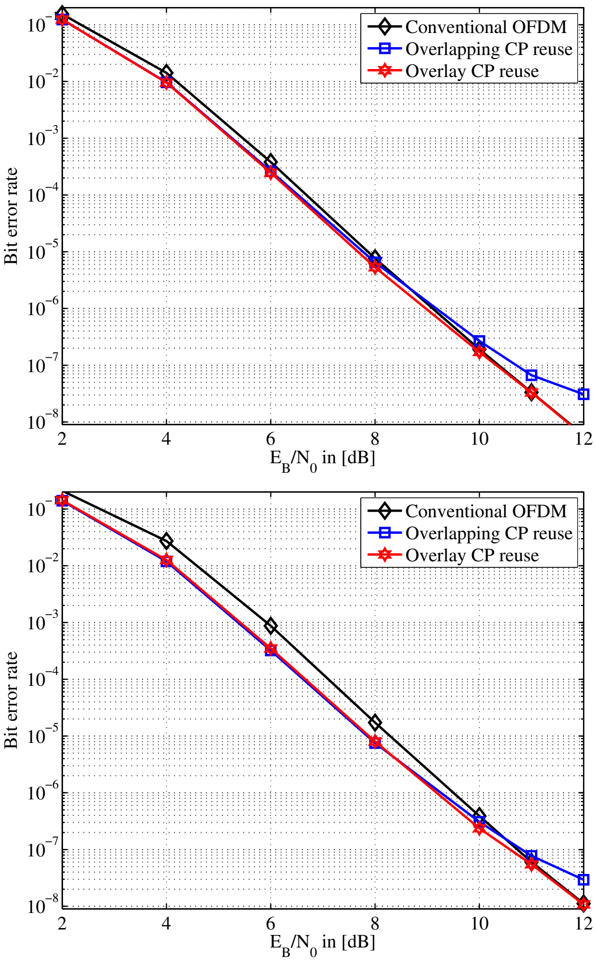

Fig. 5. BER of conventional OFDM and OFDM with CP reuse for $N_s = 2048$ with $N_{cp} = 256$ (top) and $N_{cp} = 512$ (bottom).

For larger E_b/N_0 the improvement due to noise averaging will be countervailed by the remaining equalization error of the overlapping FDE. This behavior is more significant for the overlapping CP reuse approach, as several blocks are required to equalize one OFDM symbol. However, as the main benefit of CP reuse is in the lower E_b/N_0 regions, this behavior is not a limitation.

VI. COMPUTATIONAL COMPLEXITY

The complexity of all three considered OFDM transmission systems is dominated by the FFT and the inverse FFT respectively. With $\frac{N}{2}\log_2 N$ the number of complex multiplications required to compute one FFT, the total number of multiplications per symbol necessary for a conventional OFDM system normalized to N_s is given by

$$M_{OFDM} = 1 + \frac{1}{2}\log_2 N_s. \tag{10}$$

Due to overlapping in the first approach, more data blocks have to be processed by the equalizer, so the total number of multiplications is given by

$$M_1 = \frac{1}{2}\log_2 N_s + \frac{1+\frac{N_{cp}}{N_s}}{1-\frac{2D}{N_B}}(\log_2 N_B + 1). \tag{11}$$

The overlay CP reuse requires equalization of the CP part for each symbol. This sums up to

$$M_2 = 1 + \log_2 N_s + \frac{N_B}{N_s}(\log_2 N_B + 1). \tag{12}$$

Table II shows the number of required multiplications for different ratios N_{cp}/N_s. The symbol length is set to $N_S = 2048$, the equalizer parameters N_B and D are chosen according to Section V.

TABLE II

MULTIPLICATIONS PER SYMBOL WITH $N_s = 2048$ AND DIFFERENT RATIOS N_{cp}/N_s.

N_{cp}/N_s	**Conv. OFDM**	**Overlapping**	**Overlay**
1/4	6.50	29.50	17.50
1/8	6.50	25.30	14.50
1/16	6.50	22.50	13.12
1/32	6.50	20.35	12.50

The complexity of conventional OFDM does not depend on the chosen CP length. However, for CP reuse the ratio N_{cp}/N_s significantly influences the amount of computational overhead. For both considered approaches the complexity grows with increasing CP length, though the complexity of the overlapping approach is significantly higher compared to the overlay approach. The reason for this lies in the different concepts: While for the overlay approach only one additional block has to be computed for each OFDM symbol, several blocks are required for the overlapping approach. Furthermore, larger values for N_B and D have to be chosen for the overlapping approach to achieve similar BER performance. Using overlay CP reuse significant SNR improvements can be achieved with a reasonable increase in computational complexity by a factor of ≈ 2. Note that the computational complexity of the overlay method can be further reduced by FFT pruning methods as many FFT inputs are zero.

VII. CONCLUSIONS

In this paper overlapping FDE was used to exploit redundancy in the CP to improve data estimation in OFDM receivers. While the overlapping approach has a comparatively high computational complexity, the overlay approach can significantly improve the BER with reasonably increased computational complexity. With the proposed methods the coverage area of a wireless network (DVB-T or WLAN) can be increased due to SNR improvement especially in the low SNR region.

REFERENCES

[1] R. van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*, Artech House Publishers, 2000.

[2] IEEE, “Std 802.11a part 11: Wireless LAN medium access control (MAC) and physical layer (PHY) specifications. High-speed physical layer in the 5 GHz band,” Tech. Rep., IEEE, 1999.

[3] C. V. Sinn and J. Götze, “Computationally efficient block transmission systems with and without guard periods,” *Signal Processing, Elsevier North-Holland, Inc.*, vol. 87, no. 6, pp. 1421–1433, 2007.

[4] K. Hueske and J. Götze, “Ov-OFDM: A reduced PAPR and cyclic prefix free multicarrier transmission system,” in *Proc. IEEE Int. Symposium on Wireless Communication Systems (ISWCS)*, Siena, Italy, September 2009.

[5] G.E. Bottomley and L.R. Wilhelmsson, “Recovering signal energy from the cyclic prefix in OFDM,” *IEEE Transactions on Vehicular Technology*, vol. 57, no. 5, pp. 3205–3211, Sept. 2008.

[6] A. Tarighat and A. H. Sayed, “An optimum OFDM receiver exploiting cyclic prefix for improved data estimation,” in *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, 2003.

[7] L. Vangelista, M. Rotoloni, and A. Morello, “Improved data detection exploiting full cyclic prefix for the evolution of DVB-T,” in *Int. Wireless Communications and Mobile Computing Conference (IWCMC)*, 2008, pp. 1070–1074.

[8] K. Ruttik, “A wideband radio channel model for simulation of chaotic communication systems,” in *Proceedings of ECCTD*, Budapest, August 1997, pp. 302–305.

[9] “ETSI TR 101 190 V1.3.1, Digital Video Broadcasting (DVB): Implementation guidelines for DVB terrestrial services: Transmission aspects,” Tech. Rep., ETSI, 2008.