Long Modulating Windows and Data Redundancy for Robust OFDM Transmissions

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Abstract— **Cyclic Prefixed Orthogonal Frequency Division Multiplexing (CP-OFDM) is viewed as an attractive transmissions system for wired and wireless high speed communications. Its bit error rate performance, however, can be very sensitive to impairments like frequency offset and timing mismatch as well as ingress and impulse noise. In this paper it is shown that using long modulating windows with overlap in time and good spectral containment offers significantly higher robustness to these effects compared to CP-OFDM transmissions. Further, the effect of the manner in which redundancy is inserted into an OFDM system onto its robustness is considered. If the level of impairments exceeds a specific amount, redundant OFDM transmissions based on long modulating windows and without channel coding are shown to offer a higher robustness than coded CP-OFDM transmissions.**

Index Terms— **OFDM, timing mismatch, frequency offset, ingress, impulse noise, redundancy.**

I. INTRODUCTION

CYCLIC Prefixed Orthogonal Frequency Division Multiplexing (CP-OFDM) [1] exhibits many desired features, for instance, the efficient equalizing of broadband radio channels, that make it an attractive choice for many high speed wireless communication systems. It is therefore adopted in Digital Audio Broadcasting (DAB), Digital Radio Mondiale (DRM), Digital Video Broadcasting (DVB) and Wireless Local and Metropolitan Area Networks (WLAN, WiMAX). It is also used in wired applications like ADSL (Asymmetric Digital Subscriber Line).

However, CP-OFDM transmissions are very sensitive to frequency offset and timing mismatch [2]. They may also suffer from strong narrowband interference (ingress) and impulse noise. Since all these impairments are very common in the above mentioned practical systems, the estimation of symbol boundaries and of frequency offsets needs to be particularly accurate to be able to remove Inter Symbol Interference (ISI) and Inter Carrier Interference (ICI) adequately [3].

Modulating waveforms in CP-OFDM are complex exponentials modulated by a rectangular window. In [4] it is shown that using long windows instead, which overlap each other in time and give high spectral containment, results in multicarrier transmissions that show significantly higher robustness to impairments and can combat ingress much better than CP-OFDM. Multicarrier transmissions using long windows are also discussed in [5]-[11]. While the use of the cyclic prefix inserts redundancy into the transmissions, the multicarrier transmissions based on long windows analyzed in [4] had no built-in redundancy, i.e. the interframe interval K equals the number of subcarriers N.

In this paper we analyze the robustness to channel impairments, ingress and impulse noise of redundant multicarrier systems ($K > N$) that use long windows as proposed in [12], [13] (referred to as LW-OFDM) and compare their performance with CP-OFDM. The used windowed modulating waveforms in LW-OFDM are mutually orthogonal and permit fast implementation based on a Fast Fourier Transform [12], [14]. We further investigate how BER performance and robustness to impairments depend on the technique of how a fixed amount of redundancy is inserted. In particular, we consider CP-OFDM with rate $R = 1/2$ channel coding, and CP-OFDM and LW-OFDM which use only $1/2$ of the modulating waveforms (subcarriers), but do not involve channel coding.

II. DATA MODEL

Figure 1 shows a block diagram of a general multicarrier transmission system. The data sequence to be transmitted is subdivided into the subsequences $a_0, a_1, \ldots, a_{N-1}$, which are multiplexed in

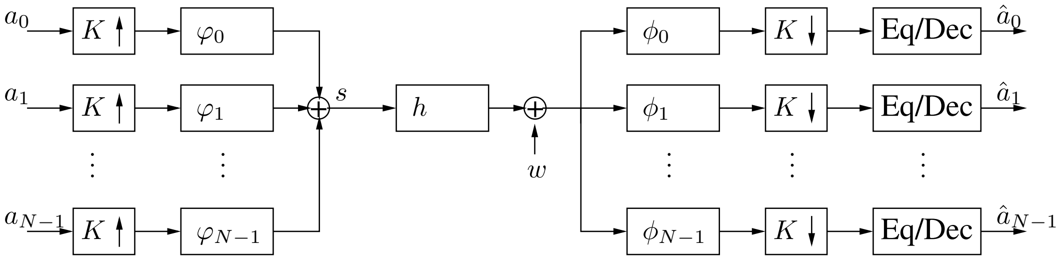

Fig. 1. Baseband block diagram of N-channel OFDM transmission system with K-point interframe interval.

frequency by using the N modulating waveforms $\varphi_0, \varphi_1, \ldots, \varphi_{N-1}$. The transmitted signal

$$s[n] = \sum_{m=-\infty}^{\infty} \sum_{k=0}^{N-1} a_k[m] \varphi_k[n - mK], \qquad (1)$$

according to Figure 1, is made up of frames that contain N symbols each and that are shifted by the interframe interval K against each other. After passing through a multipath channel $h[n]$ that is assumed to be time-invariant and after adding a sequence $w[n]$ of sampled complex Gaussian noise, N decimation filters select the information of each subchannel. The following one tap equalization and hard decision will lead to the estimates $\hat{a}_0, \hat{a}_1, \ldots, \hat{a}_{N-1}$ of the transmitted subsequences.

CP-OFDM transmissions are obtained by choosing the modulating waveforms as K-samples long complex exponentials

$$\varphi_k[n] = \frac{1}{\sqrt{N}} e^{j \frac{2\pi}{N} kn}, \quad 0 \le n \le K - 1. \qquad (2)$$

Note that $\varphi_k[n + mK] = \varphi_k[n + N + mK]$. The associated waveforms for demultiplexing are

$$\phi_k[n] = \frac{1}{\sqrt{N}} e^{j \frac{2\pi}{N} kn}, \quad -N + 1 \le n \le 0. \qquad (3)$$

Although the transmission system that uses these waveforms can be implemented extremely efficiently by applying Fast Fourier Transforms, it shows large sensitivity to impairments, such as timing mismatch and frequency offset. These impairments can be modeled by modifying the demultiplexing waveforms:

$$\phi_k[n + \Delta_t] = \frac{1}{\sqrt{N}} e^{j \frac{2\pi}{N} (k + \Delta_f) n}, \quad -N + 1 \le n \le 0. \qquad (4)$$

The integer Δ_t is the positive timing mismatch in full samples. When it is negative (in the range $[-K + N, -1]$), the interference caused by the previous OFDM symbol decreases the transmission quality only if the length of the guard period (which is $K - N$) is not longer than the channel impulse response (minus one). Therefore a negative timing mismatch might be allowed, when the duration of the guard period is chosen to be correspondingly larger than the maximal duration of the channel impulse response. Any positive timing mismatch, however, will decrease the transmission quality due to the interference caused by the following OFDM symbol. The real value Δ_f describes the frequency offset as a percentage of the subcarrier spacing. For instance, an OFDM system with a subcarrier spacing of 300 KHz and a tolerable frequency offset $\Delta_f = 1\%$ would require an oscillator accuracy of 3 kHz [1]. If those requirements are not met, additional synchronization methods need to be applied.

The use of windowed complex exponentials

$$\varphi_k[n] = v[n] \frac{1}{\sqrt{N}} e^{j \frac{2\pi}{N} kn}, \quad 0 \le n \le L_v - 1, \qquad (5)$$

where v is the modulating window and L_v is its length, has the potential to solve CP-OFDM's sensitivity problem while retaining the convenience of applying FFTs for implementation. Note, that in case of CP-OFDM the window v would be rectangular and K samples long. In [4] it has been shown that using long modulating windows can give OFDM transmission schemes that are more robust to impairments and show better capabilities to combat narrow band interference. However, the design of orthogonal windows when $K > N$, which is normally the case in OFDM transmission, is quite a challenging problem. A complete parameterization of long orthogonal windows for OFDM is given in [12], [13], where also fast implementation algorithms are proposed. In this paper we use windowed complex exponentials as proposed in [12]. The used window of length $L_v = 1024$ shows good frequency localization and is referred to as long window (LW).

III. SIMULATIONS

A. LW-OFDM vs. CP-OFDM

In Figure 2, simulated BERs of CP-OFDM and LW-OFDM based transmissions in the presence of timing mismatch and frequency offset are compared with each other. The simulations are accomplished by using a symbol rate data model. We use $N = 128$ subcarriers and an interframe interval of $K = 160$. The length of the guard period in case of cyclic prefixing would therefore be $K - N = 32$. Then, the maximum length of the channel that does not cause interference between subsequent OFDM symbols for CP-OFDM transmissions is $L = 33$ in ideal conditions ($\Delta_t = 0, \Delta_f = 0$). In the simulations a multipath channel with $L = 33$ taps that are separated by the symbol duration and whose amplitudes are Rayleigh distributed is used. The transmitted BPSK modulated data is encoded by using a convolutional encoder with code rate $R = 1/2$ and memory length $\nu = 6$. The one tap equalizer is trained by using one training frame and by assuming the absence of noise during the transmission of this frame. Timing mismatch and frequency offset, however, may occur during the training process.

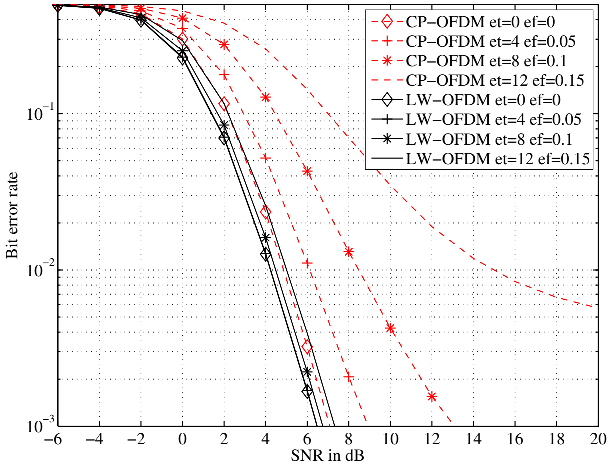

Fig. 2. Simulated BERs for CP-OFDM and LW-OFDM in the presence of timing mismatch and frequency offset. Both systems use a convolutional encoder with code rate $R = 1/2$.

In the absence of timing mismatch and frequency offset (referred to as ”et=0 ef=0” in the legend of Figure 2) LW-OFDM transmissions show little lower simulated BERs than CP-OFDM. This is due to better noise averaging behavior of the long window. In case of increasing values of impairments (referred to as ”et=4 ef=0.05”, ”et=8 ef=0.10” and ”et=12 ef=0.15”) the simulated BER of LW-OFDM transmissions remain almost identical, whereas the simulated bit error rates of CP-OFDM increase significantly. We conclude that the use of long windows can result in significantly higher robustness to frequency offset and timing offset in OFDM transmissions than the cyclic prefix approach.

B. Narrowband Interference

In addition to the scenario considered so far, a narrowband interferer with the bandwidth of one subcarrier is now placed in the middle between the 67th and 68th subcarrier. The power of the underlying Gaussian noise process of the sequence $w[n]$ is set to 9 dB. The corresponding simulated BERs of CP-OFDM and LW-OFDM transmissions as functions of different signal to interference power ratios (SIR), are presented in Figure 3. The presence of the narrowband interferer prevents reliable data transmission via subcarriers that are affected by its interference. Further, accurate estimation of the channel transfer function at the respective subcarrier frequency is impossible. In ideal conditions (”et=0 ef=0”), the subcarriers which suffer from the interferer would be lost, however, coding and frequency diversity can be used in a way that data transmissions (over the remaining subcarriers) would still work reasonably well. In the presence of timing mismatch and frequency offset, however, further subcarriers suffer from interference and the presence of the narrowband interferer diminishes the channel estimation as well as the data estimation in all further subcarriers where interference is now present as well. Robust OFDM transmissions that minimize the influence of channel impairments are therefore less vulnerable to narrowband interference. Figure 3 shows that LW-OFDM offers significantly larger robustness and significantly lower simulated BERs in the presence of a narrowband interferer than CP-OFDM. The larger the timing mismatch, frequency offset or the power of the narrowband interferer, the more the CP-OFDM’s simulated BER

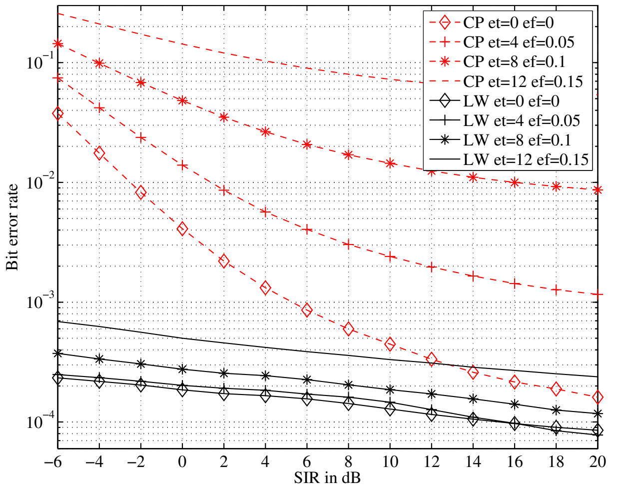

Fig. 3. Simulated BERs as function of signal-to-interference ratios for CP-OFDM and LW-OFDM in the presence of timing mismatch and frequency offset and a narrowband interferer. Both systems use a convolutional encoder with code rate $R = 1/2$.

performance suffers. LW-OFDM, on the other hand, shows only little increase of simulated BERs when impairments increase.

C. Impulse Noise

Another disturbing effect during data transmission is the presence of impulse noise, which describes high amplitude noise events which are superimposed to the Gaussian channel noise [15]. The impulse noise is modeled by regularly adding high amplitude complex noise samples to the transmitted signal. On average every 60 samples a noise burst of 2 samples length is added to the signal. For simulation we consider a transmission that is only effected by impulse noise, i.e. the power of the underlying Gaussian noise process of the sequence $w[n]$ is set to zero. The corresponding simulated BERs of CP-OFDM and LW-OFDM transmissions as functions of different signal to impulse noise power ratios (SIR), are presented in Figure 4. Analog to the Gaussian noise simulation results (see Figure 2), the BER performance using LW-OFDM is only slightly decreased, while the CP-OFDM shows significant performance drawbacks for larger impairments. Generally multicarrier transmission schemes are assumed to be more robust against impulse noise because the impulse power gets spread over all carriers in frequency domain [15]. The results show that the increased robustness of LW-OFDM compared to CP-OFDM retains in the presence of impulse noise. The lower BER of LW-OFDM is particularly explicable by the better noise averaging behavior of the long window.

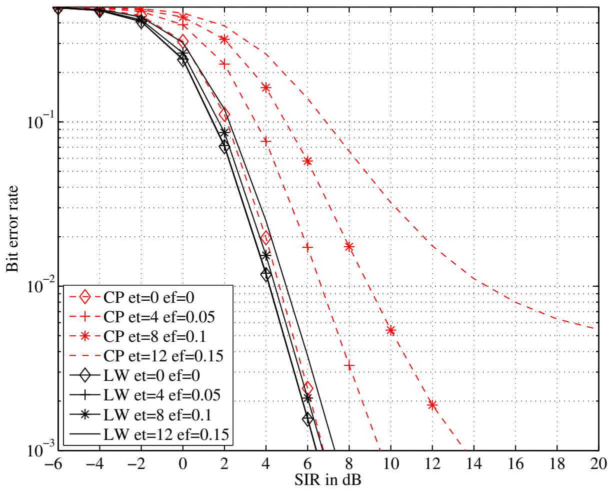

Fig. 4. Simulated BERs as funtion of signal-to-interference ratios for CP-OFDM and LW-OFDM in the presence of timing mismatch and frequency offset and additional impulse noise. Both systems use a convolutional encoder with code rate $R = 1/2$.

D. Redundancy

Beside using long modulating windows to reduce the sensitivity of CP-OFDM to impairments, the way in which redundancy is inserted might make a difference. How should redundancy be inserted into an OFDM transmission system so that the sensitivity of the transmission to timing mismatch and frequency offset is minimized? The decrease in BER performance with increasing impairments of CP-OFDM transmissions results from intercarrier interference and intersymbol interference that is not properly eliminated. Redundancy needs to be inserted in a way that this effect is minimized. One approach is the use of a convolutional encoder (like in the previous simulations) that aims at removing bit errors caused by impairments and noise. Another approach would be to use only every other subcarrier (only the even or the odd ones) and to alternate the used subcarriers with each frame. In this way, we would insert the same amount of redundancy, which is introduced by a convolutional encoder with code rate $R = 1/2$, and decrease the effect of ISI and ICI. To compare the concepts, we simulate the following three options (all with presence of impairments, but without ingress and impulse noise):

1) CP-OFDM, Code Rate $R = 1/2$

2) CP-OFDM, no coding, use only either even or odd subcarriers alternating with each frame

3) LW-OFDM, no coding, use only either even or odd subcarriers alternating with each frame

The simulated BERs for these cases with different degrees of impairments are depicted in Figure 5. For

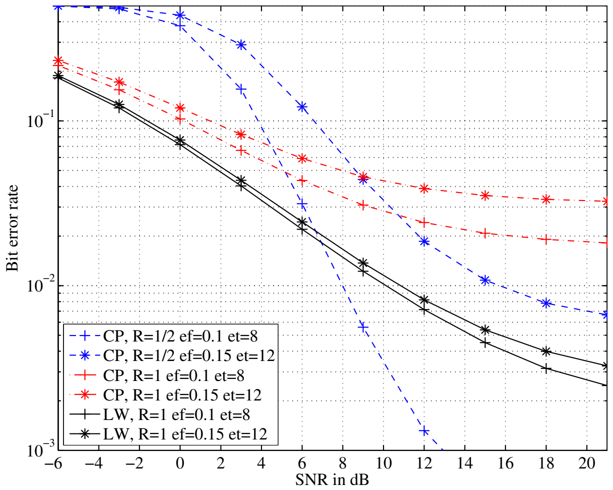

Fig. 5. Simulated BERs for CP-OFDM and LW-OFDM in the presence of timing mismatch and frequency offset (no ingress). The CP-OFDM system uses either a convolutional encoder with code rate $R = 1/2$ or uses every other subcarrier. LW-OFDM uses every other subcarrier.

the cases with no coding ($R = 1$) the long window, again, offers more robustness than the cyclic prefix. The use of every other subcarrier (instead of using an convolutional code of code rate $R = 1/2$) results in lower simulated BERs for the lower range of considered SNR values. For higher values it actually depends on the amount of impairments present in the respective transmissions. When using only the odd or even subcarriers (instead of every combined with a convolutional encoder) each subcarrier can be transmitted with a higher power to reach a similar SNR value of the superimposed signal at the receiver. The signal-to-noise ratio per (used) subcarrier is therefore better when only every other subcarrier is used. Further, the interference of the two adjacent subcarriers is eliminated. These effects result in lower BERs of this approach for the lower range of SNR values. For higher values the convolutional encoder removes bit errors quite effectively and the better option of inserting redundancy depends on the present values of impairments. For large impairments, the use of every other subcarrier, would clearly outperform the coded transmissions over the entire considered range of E_B/N_0-values. The error floors that seem to emerge in the uncoded transmissions are a result of channel zeros at subcarrier frequencies, which cause symbol errors that cannot be corrected without additional coding. Therefore, a combined approach of introducing a minimum of coding, using every other subcarrier, and applying long modulation windows would be a reasonable choice to obtain robust OFDM transmissions.

IV. CONCLUSIONS

OFDM transmissions based either on rectangular (cyclic prefixing) modulating windows or on long windows with good spectral containment have been considered. It has been shown via simulations that the use of long modulating windows has a large potential to combat impairments like frequency offset and timing mismatch. It is also more robust in the presence of ingress. Various techniques to insert redundancy are shown to have a large impact on system's performance and robustness. For large impairments redundant uncoded OFDM transmissions that use long modulating windows outperform coded CP-OFDM.

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