Improved Parameter Estimation under Non-Stationary Circumstances using Segmentation and Back Projection

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Common *phasor measurement units* (PMUs) comply with IEEE Standard C37.118 and are capable of managing the test cases formulated inside. These mainly describe electro-mechanic processes (besides step tests). However, electro-magnetic processes are not considered, but filtered out. Using subspace-based estimation algorithms provides knowledge of electro-magnetic components within the measurement signal. In addition, the system frequency can be isolated easily. The challenge of (sliding-) window-based algorithms — subspace-based as well as DFT-based methods — is a) finding a trade-off between accuracy and delay (i.e. window length) and b) the inherent assumption of (quasi-) stationarity. In this paper, the concepts of segmentation and back projection are presented which counter these challenges under the assumption of time-critical, time-stamped measurements but not time-critical delivery.

***Index Terms*—ESPRIT, OPAST, Parameter Estimation, Segmentation, Power System Measurement, Transient Processes, Back Projection**

I. Introduction

Modern power system operation requires sophisticated monitoring and analysis since the power distribution policy changes from unidirectional central power generation to a highly meshed grid of distributed and varying generation. PMUs are widely recognized for their accurate synchro-phasor measurement subject to dynamic conditions. However, only stationary signals and modulated signals with low frequency components (i.e. mainly *electro-mechanic* transients) and step tests are considered, complying with the requirements of the IEEE Standard [1] for synchro-phasors. *Electro-magnetic* transients e.g. induced by switchings in the network are not considered [2].

Since these electro-magnetic disturbances are likely to be present in a measurement but up to now undesired, PMUs commonly use filter structures to work around this problem. However, not all of these electro-magnetic transients can be filtered out by notch filter because their frequencies are not constrained to bands distant enough to the system frequency (50Hz or 60Hz, depending on the region) [3]. However, these electro-magnetic disturbances become more common and serious these days due to frequent load changes and according switchings as well as the presence of more power electronics.

Waveform analysis is a central part of this monitoring and analysis process. Yet, waveform analysis can only be done using sampling windows [4]. In consequence, there are unavoidable delays depending on the sampling frequency and the window length. In addition, a small event that is only influencing one sample compromises every window using this sample (considering a sliding window approach).

Moreover, an estimation of such an event might have a delay of at least e.g. ≈17ms (for 128 samples at a sampling frequency of 7.5kHz). Therefore, three different cases can be distinguished:

- the analysis is not as time-critical as 20ms (i.e. one *jiffy*),
- the analysis is not only time-critical in terms of accurate measurement but also in terms of delivery (i.e. minimized delay)
- the analysis is time-critical in terms of accurate measurement but its delivery may be delayed (i.e. time-stamps and a certain time of delivery are sufficient to comply with on-line requirements).

In the first case, no additional measures are necessary. For the second case, only measures resulting in a smaller window (in temporal terms, i.e. higher sampling frequency or less samples) may help. The third case may be countered by appropriate post-processing and the usage of a priori knowledge. Such post-processing will be presented in the following.

As a framework for this paper, the waveform analysis is done by subspace extraction in order to enable the detection and estimation of electro-magnetic parameters aside the system's fundamental frequency. The subspace-based parameter estimation algorithm *ESPRIT* [5] is supported by a subspace tracker of the *PAST*-family [6]. Further (complex) amplitude estimation is done with the help of a *Least-Squares* [7] approach. The extracted parameters are gathered and rated by the *DaPT* algorithm [8], [9].

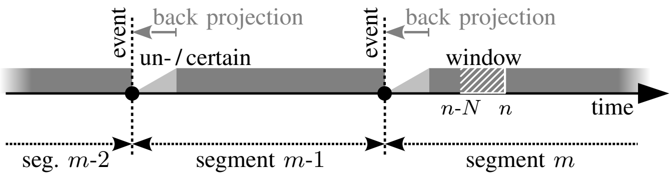

Figure 1: *Quasi*-stationary segments of measurement samples, uncertainty regions, sample window and back projection concept.

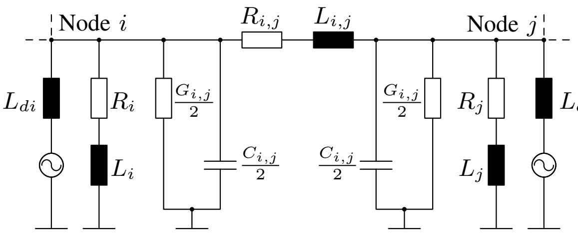

Figure 2: PI-modeled transmission line between nodes i and j; using symbol ⏦ for generators.

Most algorithms for waveform analysis require the data window to contain a stationary signal at least regarding the samples affecting the current calculation. In consequence, characteristic numbers being generated as byproducts of the waveform analysis are also affected if this requirement is not fulfilled. In the context of the above mentioned algorithms, such characteristic numbers are

(1) the estimated noise level within the subspace tracker (see Sec. III-A),
(2) the quality indices of the DaPT parameter evaluation (see Sec. III-C), and
(3) the MSE of a reconstructed signal compared to the sampled data (see Sec. III-D).

Table I: Characteristic numbers for model match

Monitoring these numbers enables a segmentation of the signal into *quasi*-stationary signal segments. Within these segments, the stationarity constraint can be fulfilled fair enough. In addition, the knowledge of the exact points of segmentation allows the post-processing of estimates near the instant of segmentation by only incorporating information from the correct segment. Due to the principle of causality, the most recent sample of a window defines the instant of time; the rest of a window is history. In consequence, near-border estimates only have to be corrected backwards. We call this *back projection*. Fig. 1 visualizes this concept.

In the following section, the signal model will be explained. The framework including ESPRIT, PAST, DaPT and LS-based amplitude estimation are reviewed in Sec. III. That section's final paragraphs introduce the methods of segmentation and back projection. A simulation demonstrates the application of these methods in Sec. IV. This paper is completed by a summary of the work and the results in a final section, also providing a glance at future work.

II. Line Modeling and Signal Model

The transmission net can be modeled as a circuit of resistances, inductances and capacitances with partly overlaid meshes, see Fig. 2 (and also Fig. 3). Each mesh with inductances and capacitances taken from the circuit has its own resonance frequency and can be excited by a switching event. In such case, each participating mesh in a node provides its resonance frequency as a superposed sinusoid in the voltage/current signal of this node.

These sinusoids appear during a transient process when the transmission net merges from one steady-state condition into another. A resulting signal model may be written as follows:

$$\begin{aligned} x(n) &= \sum_{i=1}^{p} a_i(n) e^{j\left(n\omega_i f_s^{-1} + \varphi_i\right)} + w_{\text{awgn}}(n) \\ &= \sum_{i=1}^{p} c_i(n) e^{jn\omega_i f_s^{-1}} + w_{\text{awgn}}(n) \qquad (1) \end{aligned}$$

The input samples $x(n)$ may also be used in the form of a vector $\boldsymbol{x}$ describing a set of subsequent samples. The number of sinusoids (*rank*) is p. The complex amplitude $c_i(n) = a_i(n)\exp(j\varphi_i)$ is built of the amplitude $a_i(n)$ and the (initial) phase φ_i. $\omega_i = 2\pi f_i$ describes the angular frequency. The component $\{c_1, \omega_1\}$ represents the fundamental frequency. The resonance frequency of an excited LC-oscillation mesh is embodied in $\omega_i|_{i>1}$. The components' fading-out is not modeled explicitly, but may be extracted from the time-varying character of the $c_i|_{i>1}$. Note that to increase comprehensibility, the signal model is reduced to an only-exponential model although real-valued measurements would require a description with sines or cosines (or — via Euler's identity — doubled exponentials).

III. Framework

A. Subspace Estimation

The concept of the presented parameter estimation was already discussed in [10], [11], [12]. The samples from the measurement are prepared with a sliding window before the respective data is fed to the subspace estimator.

The samples' space can be described by a set of vectors forming a basis. The obvious way to do this is to use windowed sample data to estimate the auto-covariance matrix (e.g. via exponentially weighted averaging [13]) and to perform an eigenvalue/-vector decomposition of this matrix. These eigenvectors $\boldsymbol{W}$ form one possible basis of the samples' space. Knowing the rank p, the signal's subspace can be extracted by selecting the vectors corresponding to the p greatest eigenvalues. Consequently, the other vectors span the noise subspace.

A less computational complex way to find a basis for the signal's subspace is depicted by the so-called subspace trackers. They do not produce eigenvectors; they provide vectors forming a basis describing the same space as a basis defined by eigenvectors does. Next to algorithms like PROTEUS [14] or YAST [15] the class of PAST-based algorithms [16], [17] is common in such context. In a former work [12], the OPAST algorithm was identified to be a good choice.

PAST-based algorithms are based on the nearly unconstrained minimization of the cost function

$$J\left(\hat{\boldsymbol{W}}\right) = E\left\|\boldsymbol{x} - \hat{\boldsymbol{W}}\hat{\boldsymbol{W}}^H\boldsymbol{x}\right\|^2 \tag{2}$$

where $\hat{\boldsymbol{W}}$ is not constrained to hold eigenvectors. The minimization is based on the idea of gradient-descent methods and incorporates exponential weighting for updating $\hat{\boldsymbol{W}}$ which increasingly better approximates the basis vectors $\boldsymbol{W}$ [16]. In exponential weighting $b(n) = \beta b(n-1) + (1-\beta)\hat{b}$, the factor β is also called *forgetting* factor. The *pseudo window length* $w_{\text{PAST}} = \frac{1}{1-\beta}$ gives an impression on the rage of samples affecting the current average.

Assuming valid results of OPAST, the following equations provide noise samples $\boldsymbol{w}$ and an estimation of the noise power σ^2:

$$\boldsymbol{w}(n) = \boldsymbol{x}(n) - \boldsymbol{W}\boldsymbol{y} \tag{3}$$

$$\hat{\sigma}^2_{\text{awgn}} = \|\boldsymbol{w}(n)\|_2^2 \tag{4}$$

with an internal PAST variable $\boldsymbol{y}$ which has elements only weakly related to eigenvalues.

B. Parameter Estimation

The selected basis vectors describing the desired signal space are fed to the subspace-based parameter estimator ESPRIT [5]. It is based on the analysis of the rotational invariance of two subsequent basis vectors. Due to the principle of rotational invariance, these vectors should only differ in a constant rotation $\exp\left(j \cdot n \cdot \Omega_i\right)\big|_{n=1}$ (with $\Omega_i = 2\pi f_i/f_s$) depending on the *eigenfrequency*, which is the desired parameter.

Remembering the signal model in Eq. (1), the signal can also be described by subspaces like

$$\mathbb{E}\left[\boldsymbol{x}\boldsymbol{x}^H\right] = \boldsymbol{S}(n) + \sigma^2_{\text{awgn}}\boldsymbol{I} \tag{5}$$

$$\text{with } \boldsymbol{x} = \boldsymbol{A}(n, \boldsymbol{\omega})\boldsymbol{c}(n) + \boldsymbol{w}_{\text{awgn}}(n). \tag{6}$$

The matrix $\boldsymbol{A}$ contains the exponential functions $\exp(j\omega_i n/f_s)$ to form the superposition in the horizontal dimension and the burst description in the vertical dimension. $\boldsymbol{S}(n)$ describes the signal subspace, which can be written as a product of the eigenvectors $\boldsymbol{W}(n)$ and the eigenvalues (on a diagonal matrix) $\boldsymbol{D}(n)$:

$$\mathbf{S}(n) = \mathbf{W}(n)\mathbf{D}(n)\mathbf{W}(n)^H. \tag{7}$$

One time-step can be described by a multiplication with a diagonal matrix $\mathbf{\Phi}$ having diagonal entries $\exp(2\pi j \cdot f_i/f_s)$:

$$\mathbf{W}(n+1) = \mathbf{W}(n)\mathbf{\Phi}. \tag{8}$$

The key idea of ESPRIT is to estimate the rotation $\mathbf{\Phi}$. Thinking of the burst dimension of $\mathbf{A}$, this rotation is also the factor between the upper $N-1$ lines of $\boldsymbol{W}$ and the lower. Due to noise, $\hat{\mathbf{\Phi}}$ will have off-diagonal elements. So an EVD is performed on $\hat{\mathbf{\Phi}}$. The frequencies can be extracted from these eigenvalues by calculating their angle. These frequencies are the input of DaPT.

C. Database-assisted Parameter Estimation — DaPT

Assuming the feed of ESPRIT to contain a few more basis vectors than the rank of the signal, these additional vectors belong to the noise subspace and are named auxiliaries [16]. In contrast to the vectors describing the signal, these will not result in constant parameter estimations within the ESPRIT algorithm. The idea of DaPT (*Database-assisted Parameter Estimation*) [11] is to rate the temporal presence of each frequency.

In every recursion, the incoming frequency estimation is mapped to the most suitable database entry. Ideally, both values match neglecting the measurement noise. For entries that have been successfully mapped, the rating is increased (up to a maximum value); for others, it is decreased (down to zero). A zero-quality-entry will be deleted. The frequency value of an entry is updated according to the drift measures' severity indexes exceeding a threshold. The measures' values are exponentially weighted over time (*double-exponential smoothing* [18]) and its severity index is updated sign-depended.

Entries with significant rating can be hypothesized as part of the desired signal and entries with a low rating are assumedly noise and to be forgotten. By this, the algorithm recognizes a change in rank by simply counting entries with significant ratings. This rank is advanced by a small number (the *auxiliaries*) and looped back to the block that provides the vectors to be fed to ESPRIT.

D. Estimation of Complex Amplitude

Phase and amplitude estimation can be obtained by a simple LS approach referring to the signal model in Eq. (5). Since time index and frequency (estimates) are known, the matrix $\mathbf{A}$ can be built. Together with the samples, the complex amplitude can be estimated. Using the same method, the synchrophasor can be determined by referencing a constant frequency — i.e. the system's fundamental frequency — and calibrating an offset-phase to adapt to the UTC-time [1].

Since the construction of the matrix $\boldsymbol{A}$ is not constrained to a fixed window length and window time offset but only needs to fulfill the LS requirement, i.e. an overdetermined system of equations, the application of back projection (see Sec. III-G below) is straight forward.

The signal model and the matrix $\boldsymbol{A}$, resp., can easily be used to reconstruct the input signal. This enables the calculation of the *MSE* (mean squared error) between this reconstruction and the original input samples:

$$e_{\text{MSE}}(n) = \frac{1}{N}\sum_{i=n-N+1}^{n}\left[x(i) - \boldsymbol{A}(i,\boldsymbol{\omega})\boldsymbol{c}(i)\right]^2. \tag{9}$$

E. Signal Processing Node

Considering the application of signal processing for power transmission systems, these algorithms can be grouped to nodes. Such node could replace a PMU or advance a net of PMUs. Providing a data-link between such nodes and time synchronization enables further processing for load-flow estimation, localizing and categorizing system events etc.

F. Segmentation

In the introduction (Table I) and in Sec. III, the characteristic values *noise level of PAST*, *quality indices of DaPT* and *MSE of estimation error* (after reconstruction) are accented.

The parameters can be examined to find segmentation time instants (e.g. resulting from a switching event). Approaches like simple thresholds or more sophisticated statistical means like $n \cdot \sigma$*-rules* can be used for this. These values can be expected to be fairly constant. If a significant violation is detected, the time instant indicating a transient process from one stationary state to another may be found. This evaluation provides a so-called *segmentation* of the signal.

Strictly speaking, segmentation enables the fulfillment of the *quasi*-stationarity requirement per segment. Since the signal does not change completely but only varies in most cases, the analysis does not have to start *from scratch* for each segment. Therefore, it is possible to set up adaptive versions of the before-mentioned algorithms, e.g. modifying the exponential forgetting factors of the subspace tracker or index manipulation factors of DaPT. This results in a fast adaption to the new situation which reduces the delay until a signal can be assumed to be quasi-stationary again.

G. Back Projection

Furthermore, with the help of this segmentation information, it is possible to reconstruct the parameters without the uncertainties of the compromised windows. When an event is detected and the segmentation is triggered, a timer is set to wait until the parameters can be expected to be certain again (see Fig. 1). With the help of these certain parameters, the estimation for the time between the event and the time-out can be redone time-reversed. Of course, this requires the input samples to be memorized back to the time instant of segmentation.

Although this enables the reparation of the corrupted estimates, the delay resulting from the sliding window principle cannot be compensated (*causality* rule of practical systems).

On the one hand, this procedure provides parameter estimation with a significant delay (window length plus time-out, if triggered). On the other hand, this procedure allows for the adequate fulfillment of the stationarity requirement for subsequent processing. Considering the application of PMUs in which measurement delivery to the concentrator station can be delayed, the time-out may fit within the allowed delay. We call this procedure *back projection*. In consequence, the resulting parameter estimation provides clean estimates both prior to (causality principle) and after (back projection) the event.

In the simulation provided in this paper, we enabled back projection for the estimation of the complex amplitude. Back projection could also be applied to the primary frequency estimation, but special care has to be taken regarding the matrix and vector sizes inside the subspace tracker.

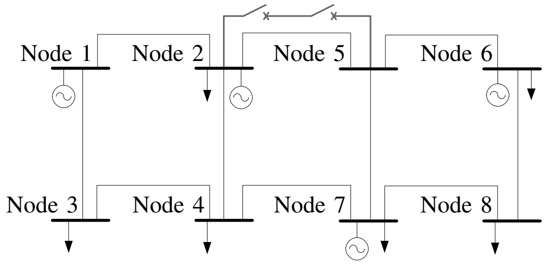

Figure 3: 8-node net to demonstrate presented concepts of segmentation and back projection; using symbols: ⏦ generator, ↓ load, and breaker.

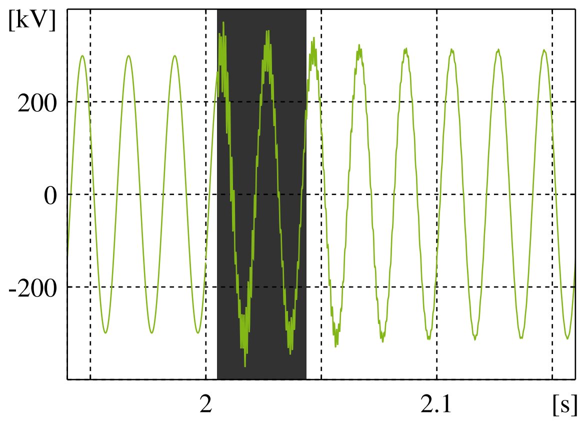

Figure 4: Cut-out of waveform samples displaying time slot containing switching and resulting electro-magnetic transients.

IV. Simulation

Considering an energy transmission system, a line between two nodes is switched during operation. In consequence, the load situation changes and (during a transient period) additional (damped) sinusoids appear in the voltage signal. These sinusoids are induced by excited LC meshes. A simulation of such scenario is done using *DIgSILENT PowerFactory*® with its simulation method for *instantaneous values*. The signal processing is done with *The Mathworks MATLAB*®. The sampling frequency is $f_s = 10$kHz; the window length is $w_l = 50$ and the OPAST forgetting factor is $\beta = 0.95$ (resulting in $w_{\text{PAST}} = (1-\beta)^{-1} = 20$ samples).

The following presents a simulation taking electro-magnetic transients into account like in Eq. (1). Within the net shown in Fig. 3, the switching of the line connecting nodes 2 and 5 happens after about 2s. Therefore, electro-magnetic transients temporarily appear before a new stationary state is reached. The resulting disturbances of the voltage signal can be seen in the highlighted region of Fig. 4.

The frequency estimation reveals the parameters of the additional sinusoids induced by excited oscillation meshes during

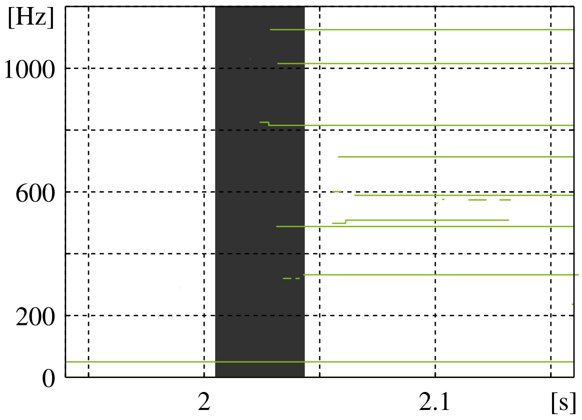

Figure 5: Frequency estimation without back projection.

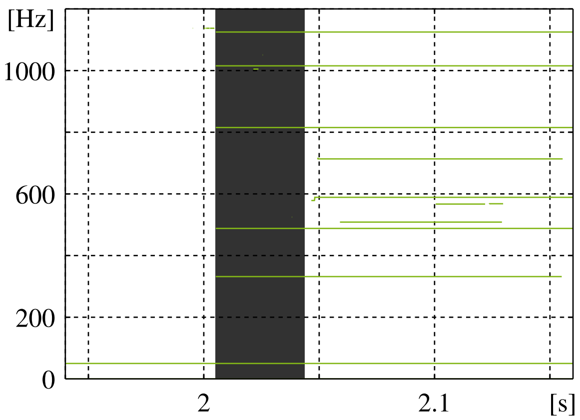

Figure 6: Frequency estimation using back projection.

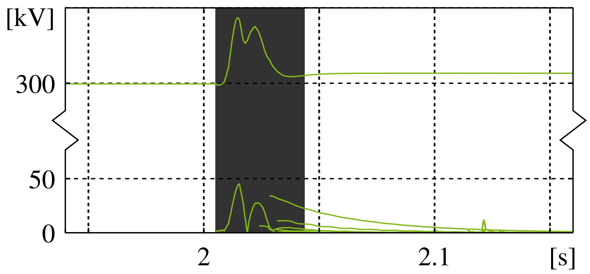

Figure 7: Amplitude estimation without back projection; plot has break in vertical scale ≷.

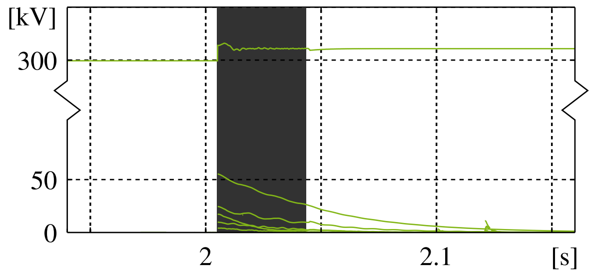

Figure 8: Amplitude estimation using back projection; plot has break in vertical scale ≷.

the transient period. Fig. 5 shows the estimation without back projection. It shall be noted, that the estimated frequencies appear fairly constant, which is because of the results of DaPT. It can be seen that not all signal components emerging after the event are strong enough to be detected within the uncertainty region.

In Fig. 6, back projection projects the estimates from the rightmost point in time of the highlighted area leftwards through this area. Doing so, a priori knowledge of the system is incorporated [2].

The amplitude estimation (as a downstream algorithm) reveals the benefit of back projection. The estimates are highly corrupted during the transient period since the parameter knowledge is not sufficient for the veridic solution of the overdetermined system of equations of the LS amplitude estimation, as can be seen in Fig. 7. Whereas in Fig. 8, back projection is incorporated in the estimation procedure and the resulting amplitude estimation becomes less corrupted near the switching event.

The three characteristic values (see Table I) all provide deviations in their course significant enough to trigger an event. However, the used $n \cdot \sigma$-rule works most reliable, if no other influences produce disturbances. Especially the varying rank produces disturbances in the noise power estimation of the subspace tracker. The selection and weighting of the quality indices of DaPT's entries is non-linear due to their temporary presence and — more challenging — their non-uniform power. This makes the MSE of the reconstructed signal compared to the input the most robust choice for segmentation.

V. Conclusion

In power system measurement applications, phasor measurement units are common. They perform very well under the framework conditions of IEEE Standard C37.118. However, the class of electro-magnetic disturbances is most commonly handled by simply filtering out these disturbances.

These electro-magnetic signal components not only cannot be constrained to reside in a frequency band distant enough to the fundamental frequency, but also provide information on the condition of the power system. Therefore, a measurement signal can be analyzed with more sophisticated signal processing algorithms like subspace-based methods, e.g. ESPRIT which is able to handle multiple sinusoidal signal components. Since most signal processing is based on windowed data, a certain delay (window length) is unavoidable. In addition, a corrupted measurement influences every window using this sample (in case of a sliding window approach).

In this paper, the concept of segmentation is presented; it monitors parameters of the incorporated algorithms like noise levels (subspace tracker), quality indices (DaPT) and/or mean squared errors (comparison of reconstructed with original signal). Since these parameters should be constant under stationary conditions, a significant step within the values of such parameter immediately indicates an event. Such information is used to split the signal into segments which are *quasi*-stationary again. The presented concept of back-projection then fills the region of still uncertain values with re-calculated estimations that only use valid samples.

The results of the shown simulation prove that the back projection of the frequency parameters back to the time instant of the switching event enables downstream processing (like amplitude estimation) to provide valid estimates again. The back projection uses the information of the prior, successful segmentation. Of course, the main prerequisite of back projection is that the frequency parameters can be regarded constant during the segments. The signal model of superposed sinusoids implies this for the ideal case (which is a stationary signal of multiple sinusoids within a segment). The segmentation performs best if the MSE of the reconstructed signal is used for triggering. Needless to say, the concept of back projection is only reasonable if the delivery of measurements is allowed to be delayed a little (but has time stamps). In case of power system measurements following the IEEE Standard C37.118, this is the case.

Future plug-ins to this work may use information from all phases of the 3-phase transmission system. In addition, the damping parameter can be incorporated explicitly. Amplitude and phase modulations (which can also be described as sinusoids very close in frequency) are still a challenge to be handled. A real-time implementation partly on FPGA is contemplated for meeting the temporal requirements of PMU measurements.

Acknowledgment

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