Database Assisted Frequency Estimation

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***Abstract*— Frequency estimation algorithms like the popular ESPRIT produce high resolution estimates. Nevertheless, there are cases where the results are not reliable, i.e when a frequency has low amplitude and is partly masked (in sense of time or spectrum). In consequence this frequency is not consistently estimated over time, meaning a noisy or fragmented output. We propose an algorithm which rates the estimations and tracks them temporally. By servicing a database holding soft-information for a frequency component we are able to produce consistent and smooth estimates, which e.g. enable reasonable phase and amplitude estimations.**

***Index Terms*—ESPRIT, PASTd, Rank Estimation, Frequency Estimation, Database**

I. INTRODUCTION

Information technology and signal processing are becoming more and more relevant in various fields of engineering. As an example, modern power grids supporting distributed generation of environmentally friendly energy and energy trading across a continent need signal processing for real-time monitoring of parameters (e.g. frequencies/phasors) and automated control.

Many frequency estimators give good results when the signal components are well separated, have similar amplitudes, and the signal-to-noise ratio (SNR) is high. However, for critical cases the performance of these estimators decreases. Some typical estimators are based on the AR(/)MA [1] noise shaping models or the model of superposed sinusoids. The latter in combination with separation of signal and noise subspaces by eigenvalue decomposition (EVD) of the correlation matrices leads to the popular MUSIC [2] and ESPRIT [3] algorithm. In a non-stationary environment ESPRIT already gives better results when replacing the traditional EVD by an eigenvalue tracking algorithm like PASTd [4], [5] (which mainly reduces computational complexity).

In addition another problem emerges when considering a temporally changing number of frequencies. ESPRIT for example requires this number to be known a-priori. To accomplish this demand the number of frequencies can be estimated by rank estimating algorithms like AIC, MDL or MAP (statistical, see e.g. [6], [7], [8]) or more sophisticated ESTER [9] or SAMOS [10]. Most of them are not very robust for weak signals either.

Here, we present an intermediate processing method which tracks multi-tone frequencies by servicing a database in order to improve the results of the frequency estimation methods (PASTd-ESPRIT is used as an example). Our method is based on the assumption that a signal is present for some time and does not change abruptly. Each database entry has two field-pairs containing frequency and frequency drift as well as a quality/severity index for both of the two values. The drift field tracks changes of the frequency over time and adjusts the frequency field if necessary. Quality index fields mark the statistical relevance of the corresponding data field, which is defined by thresholds. In the frequency context relevance means signal presence; in the other case relevance indicates possible estimation improvement. The quality index increases when a signal is present (in case of the frequency field). The severity index increases when the deviation is consistent (in case of drift). They decrease when the corresponding criterion does not match. Increase and decrease can be weighted differently to e.g. penalize the absence of estimation. Quality indices may be reset, when a correction is triggered. Advantages of the presented method are better tracking of weak frequencies once recognized and smoother frequency estimations enabling good phase/amplitude estimations. Disadvantages are the delay induced by this intermediate step and the facts that thresholds have to be reached for change.

The rest of this paper is organized as follows. First, we introduce the superposed sinusoids signal model in subsection II-A, before we briefly describe PASTd (II-B) and ESPRIT (II-C) as the main algorithms before processing, and a Kalman filter (II-E). In more detail, the database will be presented in subsection II-D. Before concluding in IV, we show some simulational results in III.

II. PROCEDURE

A. Signal Model

The model for frequency estimation embodies superposed sinusoids characterized by different frequencies, amplitudes and phases with additive white Gaussian noise (AWGN). In our examination the model order or rank is defined as the number of sinusoids p. Let

$$x(n) = \sum_{i=1}^{p} \tilde{c}_i(n) e^{jn\omega_i(n)+\phi_i} + w_{\text{awgn}}(n)$$
$$= \sum_{i=1}^{p} c_i(n) e^{jn\omega_i(n)} + w_{\text{awgn}}(n) \quad (1)$$

be samples of a measured noisy signal with symbols $c_i(n)$ being the complex amplitude ($\tilde{c}_i(n)$ the real magnitude) and $w_{\text{awgn}}(n)$ the noise. The frequency of the ith sinusoid is defined by $\omega_i(n) = 2\pi \cdot f_i(n)$. Since we interpret the phase to be constant the frequency has to be time-depended like the amplitude as we are targeting the non-stationary case. In short for the complex root of unity we write $a(n\omega_i) = e^{jn\omega_i}$ and $\mathbf{a}(n\omega_i) = [a(n\omega_i), \ldots, a((n-M+1)\omega_i)]^T$. The sample window's length is M. We define a $(M \times p)$ matrix $\mathbf{A}$ holding a column for each component:

$$\mathbf{A}(n\boldsymbol{\omega}) = [\mathbf{a}(\omega_1), \ldots, \mathbf{a}(\omega_p)] = \begin{bmatrix} e^{jn\omega_1} & \cdots & e^{jn\omega_p} \\ e^{j(n-1)\omega_1} & \cdots & e^{j(n-1)\omega_p} \\ \vdots & \ddots & \vdots \\ e^{j(n-M+1)\omega_1} & \cdots & e^{j(n-M+1)\omega_p} \end{bmatrix} \quad (2)$$

M subsequent samples form the input vector $\mathbf{x}(n) = [x(n-M+1), x(n-M+2), \ldots, x(n)]^T$ (noise vector $\mathbf{w}_{\text{awgn}}(n)$ likewise). Therefore, $\mathbf{x}(n)$ can be rewritten as

$$\mathbf{x}(n) = \mathbf{A}(n\boldsymbol{\omega})\mathbf{c}(n) + \mathbf{w}_{\text{awgn}}(n) \quad (3)$$

with vector $\mathbf{c}(n) = [c_1(n), \ldots, c_p(n)]^T$.

B. PASTd - Subspace Tracking

PASTd is short for *Projection Approximation Subspace Tracking with deflation*. The concept of subspace tracking implies a recursive approach. The underlying model (for outer recursion) is an exponential decrease, see eq. (5). The inner recursion removes the dominant component from the working vector, which initially is the input vector, until all signal eigenvalues and eigenvectors have been extracted.

With the ith eigenvector from the $(n-1)$th recursion, $\mathbf{q}_i(n-1)$, a temporary pseudo-eigenvalue y_i is generated by

$$y_i(n) = \mathbf{q}_i(n-1)^H \mathbf{x}_i(n). \quad (4)$$

The actual eigenvalue is then updated with the forgetting-factor β constituted by

$$d_i(n) = \beta d_i(n-1) + \|y_i(n)\|^2. \quad (5)$$

The corresponding eigenvector $\mathbf{q}_i(n)$ is updated by

$$\mathbf{q}_i(n) = \mathbf{q}_i(n-1) + [\mathbf{x}_i(n) - \mathbf{q}_i(n-1)y_i(n)]\,\frac{y_i(n)^*}{d_i(n)}. \quad (6)$$

The last equation removes the projection of $\mathbf{x}_i(n)$ onto the ith eigenvector ($y_i(n)\mathbf{q}_i(n)$) from $\mathbf{x}_i(n)$

$$\mathbf{x}_{i+1}(n) = \mathbf{x}_i(n) - \mathbf{q}_i(n)y_i(n). \quad (7)$$

The new $\mathbf{x}_{i+1}(n)$ now has a new dominant (formerly second dominant) eigencomponent, which will be extracted in the next step. When the recursions are done we have eigenvalues $d_i(n)$ and eigenvectors $\mathbf{q}_i(n)$ containing the current p estimated eigenvalues and eigenvectors. When calculating more eigencomponents than signal components present, the additional results are called auxiliary (/noise) eigencomponents. We can use the calculated eigenvectors as input for the ESPRIT algorithm.

Since PASTd has to know the number of eigencomponents to be extracted, a rank estimation is necessary. According to [11], this can be done by first calculating as many components as were needed (including some auxiliaries as candidates in case of increased rank) in the previous instant and then let a rank estimator decide whether to use auxiliaries, reject calculations or just move on.

C. ESPRIT - Frequency Estimation

The idea of ESPRIT is investigating the rotational invariance of two temporal sequent estimates of the signal subspace. Consider the autocorrelation matrix of the input data

$$\mathbf{R}_{xx}(n) = E\left[\mathbf{x}(n)\mathbf{x}(n)^H\right] = \mathbf{A}(n\boldsymbol{\omega})\mathbf{P}(n)\mathbf{A}(n\boldsymbol{\omega})^H + \sigma^2_{w,\text{awgn}}\mathbf{I} = \mathbf{Q}(n)\mathbf{D}(n)\mathbf{Q}(n)^H + \sigma^2_{w,\text{awgn}}\mathbf{I}, \quad (8)$$

with $\mathbf{P}(n)$ the autocorrelation matrix of the complex amplitude components of the signal. $\sigma^2_{w,\text{awgn}}$ describes the noise power. $\mathbf{Q}(n)$ and $\mathbf{D}(n)$ consequently hold the eigenvalues and eigenvectors (see PASTd). Now consider the cross-correlation matrix (with $\mathbf{y}(n) = \mathbf{x}(n+1)$)

$$\mathbf{R}_{xy}(n) = \mathbf{Q}(n)\mathbf{D}(n)\boldsymbol{\Phi}(\boldsymbol{\omega})\mathbf{Q}(n)^H + \sigma^2_{w,\text{awgn}}\mathbf{Z}, \quad (9)$$

with $\boldsymbol{\Phi}(\boldsymbol{\omega}) = \text{diag}(e^{j\omega_1}, e^{j\omega_2}, \ldots, e^{j\omega_p})$ describing a rowshift in $\mathbf{A}(n\boldsymbol{\omega})$. This shift also applies for eigenvectors $\mathbf{Q}(n)$ since both terms span the same space (signal subspace). This fact is called *rotational invariance*. Note that $\mathbf{Z}$ is related to $\mathbf{I}$ as the diagonal ones are shifted to the second diagonal.

The goal is to estimate the frequencies (inside $\boldsymbol{\Phi}(\boldsymbol{\omega})$) without knowing $\mathbf{A}(n\boldsymbol{\omega})$. We define two new symbols $\mathbf{Q}_u(n)$, which is $\mathbf{Q}(n)$ without the bottom row, and $\mathbf{Q}_l(n)$, which is $\mathbf{Q}(n)$ without the top row. Due to the rotational invariance the following equation is valid:

$$\mathbf{Q}_l(n) \approx \mathbf{Q}_u(n) \cdot \boldsymbol{\Phi}(\boldsymbol{\omega}). \quad (10)$$

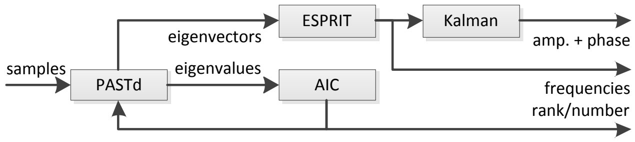
Finding the elements of $\boldsymbol{\Phi}$ is equivalent to evaluating the following least squares problem.

$$\boldsymbol{\Phi}_{\text{LS}}(\boldsymbol{\omega}) = \min_{\boldsymbol{\Phi}} \|\mathbf{Q}_u(n) \cdot \boldsymbol{\Phi}(\boldsymbol{\omega}) - \mathbf{Q}_l(n)\|_2 . \quad (11)$$

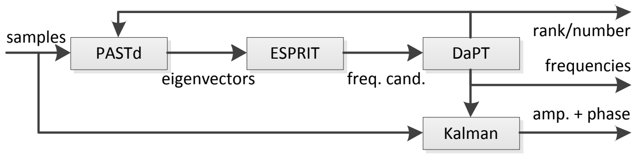
Finally, the frequencies can be extracted from the eigenvalues of $\boldsymbol{\Phi}_{\text{LS}}(\boldsymbol{\omega})$ by calculating their angle.

D. DaPT - Database assisted Parameter Tracking

Our database holds entries with four fields. In the first step, we iterate through the valid database entries sorted by the frequency's (or more general: parameter's) quality index. If the entry's frequency can be found within some uncertainty margin within the current estimation, the frequency quality, its drift and the drift's severity fields are updated. The drift's update is done by exponential windowing with a forgetting factor β. This exponential window has a pseudo length of $l_w = \frac{1}{1-\beta}$.

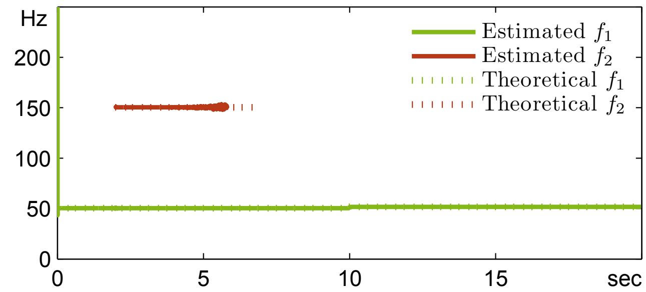


(a) Typical chain

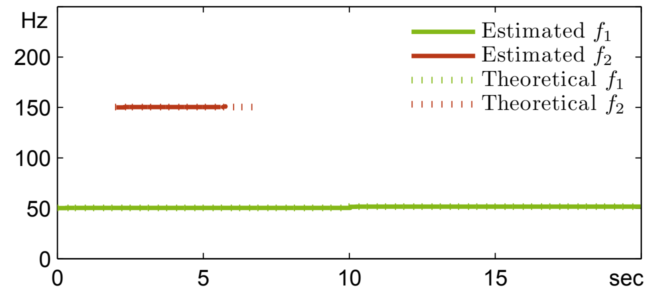


(b) New chain with database

Figure 1: possible chain of algorithms



(a) PASTd + ESPRIT



(b) PASTd + ESPRIT + DaPT

Figure 2: Frequency estimation with theoretical values given as dotted line: chain one is inaccurate in the fading region

Now, the drift severity is checked whether the frequency should be corrected. In the end, the found frequencies are marked processed. If the entry's frequency is not found, the corresponding quality index will be decreased. If this index reaches zero the entry will be deleted. If more than one estimate are similarly close to an entry only the most closest one is processed. The others are kept for the upcoming processing step.

The following step processes all not yet processed frequencies from the current estimation set. An entry will be created for each of them and its fields will be initialized to all-zero, except the frequency and its quality index, which is initialized with a small non-zero value.

To avoid the frequency field swinging due to drift-correction, the drift-correction never corrects the full drift, but a rather high portion. This can be compared to the offset of a proportional controller (P) in control engineering. This portion, the pseudo window length of the drift's exponential window and the severity thresholds influence the delay of adaption and the uncorrectable offset. If the offset is significant it will lead to a non-constant linearly in-/decreasing phase.

After issuing the relevant frequencies they can be counted. We can exchange the rank estimation between PASTd and ESPRIT by this number on the condition that we let PASTd/ESPRIT calculate some auxiliary frequencies, which are either candidates for new frequencies or noise. It is known, that the quality of ESPRIT estimates is reduced, if not the correct number of eigenvectors is fed into (see e.g. [9]), but the database can compensate that.

E. Kalman Filter for Phase and Amplitude Estimation

For estimating phase and amplitude, consider a vector containing the estimated frequencies like

$$\left[e^{j2\pi\cdot\frac{f_1}{f_s}}, e^{j2\pi\cdot\frac{f_2}{f_s}}, \ldots, e^{j2\pi\cdot\frac{f_p}{f_s}}\right].$$

In addition, consider the current position of the samples inside the input vector $[n \ldots n+M]$. Joining both equations brings a matrix ($\mathbf{A}(n)$), see subsection II-A) with one dimension for the index and one for the frequencies.

The desired complex amplitudes (*phasors* in energy engineering) written in a vector

$$\left[\tilde{c}_1 \cdot e^{j\cdot\phi_1}, \tilde{c}_2 \cdot e^{j\cdot\phi_2}, \ldots, \tilde{c}_p \cdot e^{j\cdot\phi_p}\right]^T = \mathbf{c}$$

are connected to the input samples like $\mathbf{A}(n) \cdot \mathbf{c} = \mathbf{s}(n) = \mathbf{x}(n) - \mathbf{w}(n)$. This could be solved by a least squares estimation of $\mathbf{c}$, but for better results a Kalman filter should be used.

In terms of Kalman filtering, $\mathbf{A}(n)$ is called observation matrix. The states are the complex amplitudes $\mathbf{c}$. The state transition matrix is an identity matrix.

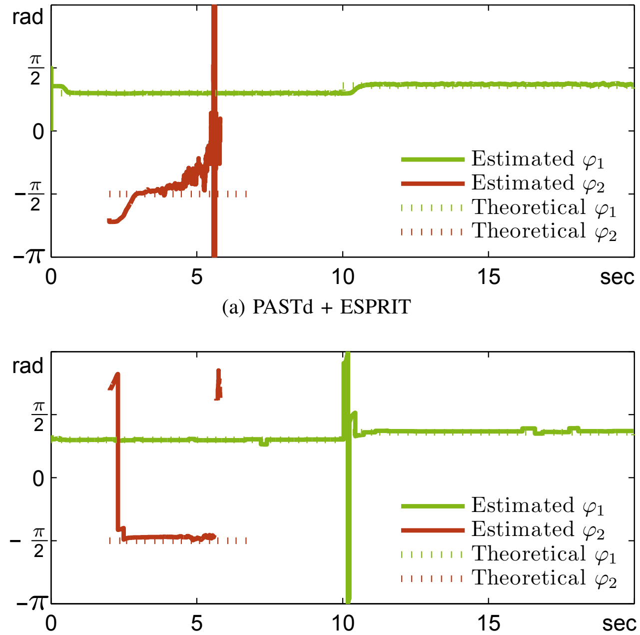
III. SIMULATION

To demonstrate the benefits, we create a test signal which is composed of a constant signal component ($230V@50,3Hz, \varphi = 0.15$), which is varied after $10sec$ to $220V@51.7Hz, \varphi = 0.18$. Another component is added after two seconds of the simulation's time ($200V@150.4Hz, \varphi = -0.25$) and is decreasing exponentially immediately. The signal is superposed with white Gaussian noise. The sampling frequency is $10kHz$, the length of the simulations is $20sec$.

The simulation is done with algorithms having

1) PASTd and ESPRIT only (with subsequent Kalman filter) and
2) the additional DaPT processing in between ESPRIT and the Kalman filter.

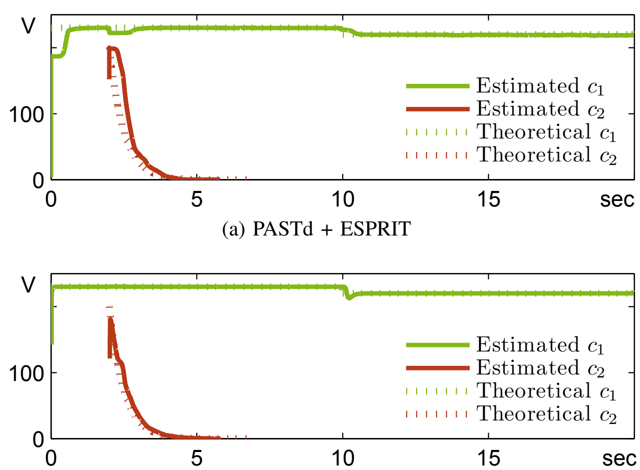
Investigating the frequency estimation, both algorithm chains produce reasonable results as can be seen in fig. 2. In the case of the fading component at about the fifth second it can be observed that chain one becomes inaccurate.



(a) PASTd + ESPRIT

(b) PASTd + ESPRIT + DaPT

Figure 3: Phase estimation with theoretical values given as dotted line: little frequency misestimations lead to severe phase misestimations



(a) PASTd + ESPRIT

(b) PASTd + ESPRIT + DaPT

Figure 4: Amplitude estimation with theoretical values given as dotted line: phase/amplitude adaption delay is higher for chain one

We created the database since the phase estimations of pure PASTd/ESPRIT were bad. This is because of the sensitivity of the phase. Little misestimations of the frequency will lead to severe phase misestimations (see fig. 3a). The phase estimation of DaPT is more reasonable as fig. 3b shows.

In the PASTd+ESPRIT-chain, we chose Kalman filter parameters that lead to a longer history than in DaPT. By this we have smoothen the phase estimation in fig. 3a a little. This is paid by an increased adaption delay. This fact can be seen in fig. 4a. In contrast, DaPT has no need for this disadvantageous choice of Kalman parameters.

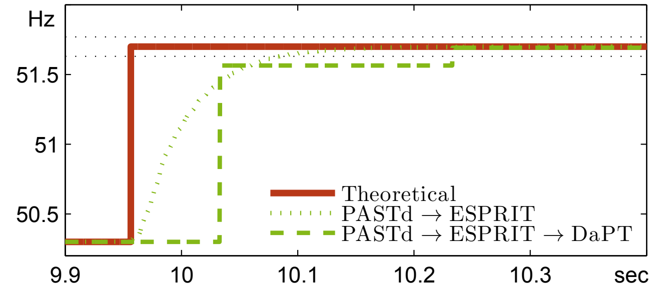


Figure 5: Adaption Delay: the signal change at the tenth second shows that DaPT needs roughly a doubled time to adapt in contrast to PASTd+ESPRIT only

The adaption time for the frequency parameter is influenced by the database since thresholds have to be reached before an adaption takes place. Fig. 5 shows that PASTd+ESPRIT only needs about $0.12sec$ to adapt to the new condition (within some uncertainty margin, dotted line). DaPT needs $0.27sec$ in this simulation, which is about twice the time. This time varies from simulation to simulation since the database is non-linearly reaching its thresholds. It hardly depends on the database parameters.

IV. Conclusion

When dealing with joint parameter estimation the quality of post-processing results like phase and amplitude estimations of a Kalman filter are strongly influenced by the quality of intermediate results like frequency estimation.

As a countermeasure, we introduced a database (DaPT – *Database assisted Parameter Tracking*), which tracks and rates these intermediate results (frequencies). It adapts its fields by monitoring the drift and the quality of the parameter. Thresholds define whether to not change or to correct the main parameter, which in our case is the frequency.

The simulations have shown that the database post-processing is able to give more accurate phase and amplitude estimations in fading cases. In other words, once detected a signal component can be tracked with good stability when it fades away. This is paid by an additional delay and some fair computational complexity.

The database approach has another benefit that was not demonstrated in this paper. The Kalman filter we used for estimating phase and amplitude needs inputs without gaps, but gaps can occur e.g. when the eigenvectors of PASTd for weak signal components are by mistake mixed up with noise. On the on hand, single calculation turns of the Kalman filter can be left out if inputs for all components are missing. On the other hand, if calculation turns are performed with incomplete input the results will be faulty. The database approach prevents this case.

As mentioned before, DaPT never corrects the full drift error to avoid swinging. We described this by the analogy of the proportional controller in control engineering. The offset that might persist is removed in control engineering by a proportional plus integral controller. With the intent of improving DaPT, the integration character can be implemented by investigating the phase difference of two subsequent estimations to extract the slope. This slope can be used to correct the database entries.

In future work, the database may be extended to also track the first derivation of the frequency which enables the tracking of linearly increasing or decreasing frequencies (thinking of DOA problems). We will also analyze the phase-difference-approach.

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